1. Consider a particle of mass m within a one-dimensional (1D) infinite potential well

$$V(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq L, \\ \infty, & \text{otherwise,} \end{cases}$$

where L is a positive constant. Suppose the particle is initially in the ground state and the right wall of the well moves suddenly from $x = L$ to $x = 2L$, while leaving the wave function of the particle undisturbed at that instant. If energy is measured for the particle:

(a) (10 points) What is the most probable result and the corresponding probability?

(b) (10 points) What is the expectation value of the energy?

2. Consider a particle confined to a ring of radius R on the xy-plane with the z-axis passing through its center. Suppose the particle has the wave function

$$\psi(\phi) = \mathcal{N}\left[1 + 2\, i \cos(3\phi)\right] ,$$

where $\mathcal{N}$ is a normalization constant, ϕ is the azimuthal angle in cylindrical coordinates, and $i \equiv \sqrt{-1}$.

(a) (5 points) Find the normalization constant $\mathcal{N}$.

(b) (10 points) Find the amplitudes c_m in the expansion

$$\psi(\phi) = \sum_{m=-\infty}^{\infty} c_m \, \Phi_m(\phi)$$

with $\Phi_m(\phi) \equiv e^{im\phi}/\sqrt{2\pi}$.

(c) (10 points) If one measures the z-component of the particle's orbital angular momentum, what are the possible results and the corresponding probabilities?

3. Consider a 1D simple harmonic oscillator (sho) of mass m and frequency ω which has the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 ,$$

where $\hat{p}$ and $\hat{x}$ are, respectively, the momentum and the position operators.

(a) (10 points) Express the Hamiltonian $\hat{H}$ in terms of the lowering operator

$$\hat{a} \equiv \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} + \frac{i\hat{p}}{m\omega} \right) .$$

(b) (15 points) Suppose at time $t = 0$ we have $\langle\, \hat{x}(0)\, \rangle = x_0$ and $\langle\, \hat{p}(0)\, \rangle = p_0$ for the sho, find $\langle\, \hat{x}(t)\, \rangle$ at any time t. (Hint: You may like to use the identity

$$\exp(\hat{B})\, \hat{A} \exp(-\hat{B}) = \hat{A} + [\hat{B}, \hat{A}] + \frac{1}{2!}\, [\hat{B}, [\hat{B}, \hat{A}]] + \cdots .)$$

(c) (30 points) If the sho carries a charge q and is under the action of a time-dependent homogeneous electric field (along the x-direction)

$$\mathcal{E}(t) = \mathcal{E}_0 \exp(-t^2/\tau^2) \cos(\omega_0 t)\,,$$

where $\mathcal{E}_0$, τ, and ω_0 are constants. Suppose at time $t \to -\infty$ the sho starts in the ground state $|0\rangle$, using first-order perturbation theory, find the probability that the sho will be found in other energy eigenstates $|n\rangle$ at time $t \to \infty$. (Hint: You may need $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a}$.)