Useful constants:
(1) $h = 6.62\times10^{-34}$ J·s, Planck constant
(2) $m_e c^2 = 0.5$ MeV, m_e: electron's rest mass, c: the speed of light.
(3) $hc = 1240$ eV·nm

Problems:

1. At the presence of a uniform magnetic field B in the z-direction, an electron travels in the x-y plane. Staring from Bohr's atomic model, derive expressions for (i) (5%) the radius and (ii) (5%) the energy of the electron's energy state. Neglect electron's spin in this problem. However, you need to consider the interaction between the magnetic moment produced by the circulating electron and the B field. Let e be the charge and m the mass of the electron in your answers.

2. A beam of electrons passes through an electric field with potential V.
(a) (5%) Find the de Broglie wavelength of the electrons.
(b) (5%) The electron beam is now aimed at a crystal. The first diffraction maximum is observed when the beam incidents at an angle θ with respect to the crystal surface. What is the spacing between lattice planes?
Express your answers in terms of V, θ, e (electron charge), m (electron mass) and other constants.

3. (10%) The maximum energy obtained by electrons in a Compton scattering experiment is 10 KeV. What is the energy of the incident photon? (To solve this problem you need to determine under what condition the electron is able to acquire maximum kinetic energy.)

4. (a) (5%) A blackbody's temperature is increased from 300 K to 2700 K. By what factor does the total power radiated per unit area increase?
(b) (5%) If a rod travels with a speed $v = 0.8\,c$ along its length relative to an observer, what is the rod's length measured by the observer?

5. (a) (5%) Some physics theories indicate that the lifetime of the proton is about 10^{36} years. What would such a prediction say about the energy of the proton?
(b) (5%) The threshold frequency of potassium is 558 nm. What is the work function for potassium? What is the stopping potential when light of 400 nm is incident on potassium?

6. The wave function of a particle trapped in an infinite square well potential of width a is found to be :

$\Psi(x) = C\sin\frac{\pi x}{a} + \frac{1}{2}\sin\frac{3\pi x}{a}$ inside the well,

$\Psi(x) = 0$ outside the well.

(a) (10%) Calculate the coefficient C.

(b) (10%) If a measurement of the total energy is made, what are the possible results of such a measurement, and what is the probability to measure each of them ?

7. (15%) A particle moving under the influence of a potential $V = \frac{1}{2}x^2$ has a wave function $\Psi(x)$. If we change the wave function to $\Psi(3x)$, by what factors are the mean kinetic and potential energies changed ?

8. (15%) Solve and classify the eigenvalues of the Hamiltonian $H = \vec{\sigma}_1 \cdot \vec{\sigma}_2$, where $\vec{\sigma}_1, \vec{\sigma}_2$ are the Pauli spin matrices for particle (1) and (2) respectively.