

國立中正大學 104 學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系

學系別：資訊工程學系、電機工程學系、機械工程學系

科目：微積分

化學工程學系、通訊工程學系

第 1 節

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一、單選題(每題 4 分, 共 16 分).

1. Which of the following must be true?

(A) $\lim_{x \rightarrow 0} \frac{x}{e^x} = \lim_{x \rightarrow 0} \frac{1}{e^x} = 1$ (B) If $g(x) = |x - 2|$, then $\lim_{x \rightarrow 2^+} \frac{g(x) - g(2)}{x - 2}$ does not exist

(C) $\lim_{x \rightarrow 0^+} \tan^{-1}(1/x)$ does not exist (D) None of the above

2. Which of the following must be true?

(A) The series $\sum_{k=2}^{\infty} \ln\left(\frac{k^2 - 1}{k^2}\right)$ converges

(B) The series $\sum_{k=1}^{\infty} \frac{(-1.1)^k}{k}$ converges absolutely

(C) The series $\sum_{k=2}^{\infty} \frac{\ln k}{k^{1.1}}$ diverges

(D) None of the above

3. Let $f(x, y) = x^3 - 12xy + 8y^3$. Which of the following must be true?

(A) $f(x, y)$ has three critical points (B) $(2, 1)$ is a saddle point of $f(x, y)$

(C) At the point $P(1, 1)$, $f(x, y)$ increases fastest in the direction $(-3/5, 4/5)$

(D) None of the above

4. Let C be a smooth curve defined by $\mathbf{r}(t)$ and C is also represented as $\mathbf{r}(s)$ in terms of the arc length parameter s . Which of the following must be true?

(A) $ds = \mathbf{r}'(t)dt$ (B) the unit tangent $\mathbf{T} = \frac{d\mathbf{r}}{ds}$

(C) the unit normal $\mathbf{N} = \frac{d\mathbf{T}}{ds}$ (D) None of the above

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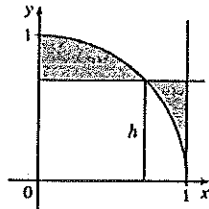
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二、填空题(每個空格 8 分, 共 64 分)

1. Let D be the region bounded by the parabola $y = x^2 - x$ and the x -axis. Find the slope of the line passing through the origin that divides D into two regions with equal area. (a)
2. The following figure shows a unit circle in the first quadrant. At what height (h) above the x -axis should the horizontal line be placed so as to minimize the shaded area? (b)



3. Find the limit: $\lim_{x \rightarrow 1^+} (\ln(x^5 - 1) - \ln(x^3 - 1))$. (c)
4. Find the area of the surface generated by revolving the cardioid $r = 1 + \cos \theta$ around the x -axis. (d)
5. Find the first three nonzero terms in the Maclaurin series for the function $f(x) = \int_0^{\pi/2} \frac{dt}{1 - x^2 \sin^2 t}$. (e)
6. Find the sum of the series: $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n} (2n)!}$. (f)
7. Find the average value of the function $f(x, y, z) = x^2 z + y^2 z$ over the region enclosed by the surface $z = x^2 + y^2$ and the plane $z = 1$. (g)
8. Let S be the part of cylinder $y^2 + z^2 = 4$ that lies above the rectangle with vertices $(0,0)$, $(1,0)$, $(1,2)$ and $(0,2)$. Find the area of the surface S . (h)

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三、計算題(20 分, 這部分必須有詳細演算過程, 僅有答案而沒有演算過程得 0 分)

1. (10 分) Evaluate $\iint_S (\mathbf{F} \cdot \mathbf{N}) dS$, where $\mathbf{F} = (xy, y^2, x^3y^3)$, \mathbf{N} is the outer normal and S is the surface bounded by the parabolic cylinder $x = y^2$ and the planes $x = z$, $z = 0$ and $x = 1$.

2. (10 分) Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C . If $f(x, y)$ is a scalar function with continuous first and second partial derivatives in D , derive the following formula:

$$\iint_D (f_{xx} + f_{yy}) dx dy = \int_C \frac{\partial f}{\partial n} ds$$

where $\frac{\partial f}{\partial n} = \nabla f \cdot \mathbf{N}$ is the normal derivative, \mathbf{N} is an outer unit normal, s is the arc-length.