(1) ( 15 % ) Two particles , each of ( rest ) mass $m$ , collide head-on at velocity $\frac{4}{5}c$ . They stick together. What is the mass of the composite object ?

(2) ( 15 % ) Express ( a ) Fine structure constant, ( b) Classical radius of electron, ( c ) Electron Compton wavelength, in terms of the fundamental constants $e, m_e, c,$ and Planck constant.

(3) ( 10 % ) What is the ground state energy of an atom consisting of an electron and a positron bound to each other by their coulomb forces ?

(4) ( 10 % ) A typical atomic nucleus is about $5.0\times10^{-15}m$ in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus.

(5) Consider a non-interacting electron gas confined in the potential well

$$V(x,y,z) = \begin{cases} 0 & \text{for } |z| < a\,, \\ \infty & \text{for } |z| \geq a\,, \end{cases}$$

where $a$ is a positive constant. Denoting the mass of the electron $m$:

(a) (10 %) Find the eigenfunction $\psi(x,y,z)$ for a single electron.

(b) (15 %) Find the eigenenergy of the electron and make a plot for the eigenenergy versus the parallel component of the wavevector, ie. $k_{\parallel} \equiv \sqrt{k_x^2 + k_y^2}$.

(c) (15 %) Suppose the Fermi energy of the system is

$$E_F = \frac{\pi^2 \hbar^2}{ma^2}\,,$$

find the electron density of the system at temperature $T$.

(6) (10 %) Consider a sodium atom in a magnetic field. If spin-orbit interactions are taken into account, how many different energy levels would the $n = 2$ level split into?