1. For the atom of Carbon (C), its atomic number is 6 and with an electronic configuration like $(1s)^2(2s)^2(2p)^2$.
(a) (5%) What are total spin S and total orbital angular momentum L for the two $2p$ electrons?
(b) (3%) What are the possible spectroscopic notations for the states if we don't consider the anti-symmetric property of the wave functions for the "identical" electrons?
(c) (3%) What are the possible spectroscopic notations for the states if the anti-symmetric property of the wave functions is considered?
(d) (3%) Using Hund's rule to determine the ground state of the carbon atom.

2. (a) (6%) Write down, and briefly explain, **three** experiments showing the evidences of energy quantization in an atom.
(b) (3%) What is the "exchange force"? Where is the "exchange force" coming from?

3. We have N (N >>1) identical fermions of spin 1/2 which are trapped in a one-dimensional potential region $0 < x < D$, between two infinite walls. Assume the mass of each fermion is m. The interacting potential between the fermions is 0.
(a) (3%) What is the degeneracy for each energy level n for the N spin=1/2 fermions? **Note**: the lowest energy level corresponds to n=1.
(b) (3%) What is the N-particle ground state energy, i.e., the lowest **total** energy for the N particles in the well. (useful formula: $\Sigma_{j=1}^{n} j^2 \sim n^3/3$, if $n >> 1$).
(c) (3%) What is the Fermi energy E_F when constructing the N-fermion ground state?
(d) (3%) What is the corresponding de Broglie wave length for the particles at the energy level having the Fermi energy?
(e) (3%) Estimate the degenerate pressure of the N-fermion system while in the ground energy state.

4. (a) (6%) What is the Pauli's exclusion principle? It works for both bosons and fermions? Electron is a fermion? Photon and neutron are bosons? Please **briefly** explain your answers.
(b) (6%) What is the Bohr radius a_0? How big is it? Estimate the radius, in terms of a_0, of a hydrogen atom in the **ground** state using the Bohr model.

5. Consider a rocket moving with velocity v relative to an observer in front of it. If the rocket sends out a laser pulse with speed c and frequency f towards the observer.

(a) (5 %) What is the velocity of the pulse measured by the observer?

(b) (5 %) What is the frequency of the pulse measured by the observer?

(c) (5 %) Instead of a laser pulse, the rocket now sends out a radioactive beam of atoms. If the observer detects that the atom has a lifetime τ, find the lifetime of the atom when it is **at rest**.

6. Consider a particle of mass m confined in the potential well

$$V(x) = \begin{cases} 0 & |x| < a, \\ \infty & |x| \geq a, \end{cases}$$

where a is a positive constant. Suppose the particle is in a state described by the wave function

$$\psi(x) = \begin{cases} C\,(x^2 - a^2) & |x| < a, \\ 0 & |x| \geq a, \end{cases}$$

where C is a **positive constant**.

(a) (5 %) Determine the constant C by normalizing the wave function $\psi(x)$.

(b) (5 %) If one measures the energy of the particle, what is the expectation value?

(c) (10 %) In the same measurement as in (b), what is the probability of obtaining the ground state energy?

7. (15 %) Suppose the conduction electrons in a metal can be treated as a free electron gas. If the Fermi temperature of the metal is T_F and the electron number is N, find an approximate expression for the magnetic susceptibility of the metal (denote the Boltzmann constant k_B, the Bohr magneton μ_B, and the permeability constant μ_0).