Useful constants:

1. Planck's constant $h = 6.62 \times 10^{-34}$ J·s.
2. Speed of light $c = 3 \times 10^{8}$ m/s.
3. Electron charge $e = 1.6 \times 10^{-19}$ C.
4. Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Problems:

1. A ruler of 1 meter long moves parallel to its length with speed $v = 0.6c$ relative to you.
   (a) (5 points) Compute the length of the ruler measured by you.
   (b) (5 points) How long does it take for the ruler to pass you?

2. (10 points) The energy reaching Earth from the sun at the top of the atmosphere is 1.36×10^{3} W/m^2, called the solar constant. Assuming that Earth radiates like a blackbody at uniform temperature, what is the equilibrium temperature of Earth?

3. (10 points) Consider the hydrogen atom. Assuming the angular momentum of the electron is quantized, i.e., $L = n\hbar$ with $n = 1, 2, 3, \ldots$, derive an expression for the energy of the electron. Express your answer in terms of n, $\hbar$, the electron charge e, the electron mass m, and the Coulumb constant k.

4. (10 points) If the uncertainty in the position of a wave packet representing the state of a particle is equal to its de Broglie wavelength, how does the uncertainty in momentum compare with the value of the momentum of the particle?

5. (10 points) A certain crystal has a set of planes spaced d apart. A beam of neutrons strikes the crystal at normal incidence and the first maximum of the diffraction pattern occurs at an angle θ with respect to the incident beam. What is the kinetic energy of the neutrons? Express your answer in terms of d, θ, the neutron mass m, and other constants.

6. (a) (5 points) What is the Stern-Gerlach experiment? Describe briefly the design of the experiment and the phenomenon one could observe from it.
(b) (5 points) The angular momentum of the yttrium atom in the ground state is characterized by the quantum number $j = 3/2$. How many lines would you expect to see if you perform a Stern-Gerlach experiment with yttrium atoms?

7. If the angular momentum of the nucleus is $\boldsymbol{I}$ and that of the atomic electrons is $\boldsymbol{J}$. The total angular momentum of the atom is $\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$. Denote the quantum numbers of $\boldsymbol{I}, \boldsymbol{J}, \boldsymbol{F}$ as I, J, F, respectively.
(a) (5 points) What is the range of F for fixed values of I and J?
(b) (5 points) The potassium isotope $^{40}\mathrm{K}$ has $I = 4$. What is the total number of possible F values for a state with $J = 3/2$?

8. Two noninteracting Fermions are confined in a potential well

$$V(x) = \begin{cases} \infty & |x| > a/2 \\ 0 & |x| < a/2 \end{cases}$$

For simplicity, assume that the particles have no spins (and yet are still *Fermions*!)
(a) (5 points) Find the ground state energy of the system.
(b) (5 points) What is the ground state wavefunction of the system?

9. Consider a three dimensional system of N particles which has only two possible energy states $E_1 = 0$ and $E_2 = \varepsilon$. The distribution function is $f_i = C \exp(-E_i / kT)$, where k is the Boltzmann constant.
(a) (5 points) Find the average energy $\langle E \rangle$.
(b) (5 points) Find the heat capacity of the system.

10. (10 points) The energy gap between the valence band and the conduction band in silicon is 1.14eV at room temperature. What is the maximum wavelength of a photon that will excite an electron from the top of the valence band to the bottom of the conduction band?