1. (10 points) Consider a pion π^+ that decays at rest into a muon μ^+ and a neutrino ν, i.e. $\pi^+ \to \mu^+ + \nu$. Let the mass of μ^+ be m_μ, the kinetic energy be K and assume $m_\nu = 0$. Find the mass m_π of π^+. (please express m_π in terms of m_μ, K and the speed of light c).

2. Consider a one-dimensional quantum well with potential of the form

$$V(x) = \infty \qquad x < 0, x > L$$
$$= 0 \qquad 0 < x < L$$

(a) (8 points) If the eigenfunctions are

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, \quad n = 1, 2, \cdots$$

find the energy levels E_n of the quantum well.
(b) (8 points) Suppose a particle in the above potential has an initial normalized wave function of the form

$$\phi(x, 0) = C \left(\frac{\sin \pi x}{L} \right)^3$$

then what is the form of $\phi(x, t)$?
(c) (8 points) Find the probability that an energy measurement of $\phi(x, 0)$ yields E_2.

3. The uncertainty relations may be used to make numerical estimates in microscopic physics. Please use the uncertainty relations to estimate the ground state energies of the following systems.
(a) (8 points) hydrogen atom,
(b) (8 points) one-dimensional harmonic oscillator.

6. (a) (5 points) What is the Stern-Gerlach experiment? Describe briefly the design of the experiment and the phenomenon one could observe from it.
(b) (5 points) The angular momentum of the yttrium atom in the ground state is characterized by the quantum number $j = 3/2$. How many lines would you expect to see if you perform a Stern-Gerlach experiment with yttrium atoms?

7. If the angular momentum of the nucleus is $\boldsymbol{I}$ and that of the atomic electrons is $\boldsymbol{J}$. The total angular momentum of the atom is $\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$. Denote the quantum numbers of $\boldsymbol{I}, \boldsymbol{J}, \boldsymbol{F}$ as I, J, F, respectively.
(a) (5 points) What is the range of F for fixed values of I and J?
(b) (5 points) The potassium isotope $^{40}\mathrm{K}$ has $I = 4$. What is the total number of possible F values for a state with $J = 3/2$?

8. Two noninteracting Fermions are confined in a potential well

$$V(x) = \begin{cases} \infty & |x| > a/2 \\ 0 & |x| < a/2 \end{cases}$$

For simplicity, assume that the particles have no spins (and yet are still *Fermions*!)
(a) (5 points) Find the ground state energy of the system.
(b) (5 points) What is the ground state wavefunction of the system?

9. Consider a three dimensional system of N particles which has only two possible energy states $E_1 = 0$ and $E_2 = \varepsilon$. The distribution function is $f_i = C \exp(-E_i / kT)$, where k is the Boltzmann constant.
(a) (5 points) Find the average energy $\langle E \rangle$.
(b) (5 points) Find the heat capacity of the system.

10. (10 points) The energy gap between the valence band and the conduction band in silicon is 1.14eV at room temperature. What is the maximum wavelength of a photon that will excite an electron from the top of the valence band to the bottom of the conduction band?