Useful information

1. $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$.

2. Normalized radial functions for the one-electron atom for $n = 1, 2,$ and 3.

n	ℓ	$R_{n\ell}(r)$
1	0	$2\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	$\left(\frac{Z}{2a_0}\right)^{3/2}\left(2-\frac{Zr}{a_0}\right)e^{-Zr/2a_0}$
2	1	$\frac{1}{\sqrt{3}}\left(\frac{Z}{2a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)e^{-Zr/2a_0}$
3	0	$\frac{2}{3}\left(\frac{Z}{3a_0}\right)^{3/2}\left(3-\frac{2Zr}{a_0}+\frac{2Z^2r^2}{9a_0^2}\right)e^{-Zr/3a_0}$
3	1	$\frac{2\sqrt{2}}{9}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{2Zr}{a_0}-\frac{Z^2r^2}{3a_0^2}\right)e^{-Zr/3a_0}$
3	2	$\frac{4}{27\sqrt{10}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{Z^2r^2}{a_0^2}\right)e^{-Zr/3a_0}$

3. Some normalized spherical harmonics.

ℓ	m	$Y_{\ell m}(\theta, \varphi)$
0	0	$Y_{00} = \frac{1}{\sqrt{4\pi}}$
1	0	$Y_{10} = \left(\frac{3}{4\pi}\right)^{1/2}\cos\theta$
	± 1	$Y_{1\pm 1} = \mp\left(\frac{3}{8\pi}\right)^{1/2}\sin\theta\, e^{\pm i\varphi}$
2	0	$Y_{20} = \left(\frac{5}{16\pi}\right)^{1/2}(3\cos^2\theta - 1)$
	± 1	$Y_{2\pm 1} = \mp\left(\frac{15}{8\pi}\right)^{1/2}\sin\theta\cos\theta\, e^{\pm i\varphi}$
	± 2	$Y_{2\pm 2} = \left(\frac{15}{32\pi}\right)^{1/2}\sin^2\theta\, e^{\pm 2i\varphi}$

Problems:

1. The normalized wave function for a one-electron atom is

$$\psi_{nlm}(\vec{r}) = R_{nl}(\vec{r})Y_{lm}(\theta,\varphi).$$

(a) (10 points) Find the location at which the radial probability density is a maximum for the ground state of this one-electron atom.
(b) (10 points) Find the expectation value for the radial coordinate in this ground state.
(c) (5 points) After comparing your answers in (a) and (b), explain why they are *or* are not equal.

2. Potassium (K) is an alkali metal with $Z = 19$.
(a) (5 points) What is the electron configuration of the ground state of potassium?
(b) (10 points) What are the L, S, and J quantum numbers of this state?
(c) (10 points) Describe the Zeeman effect of the ground state quantitatively.

3. (10 points) The relation between the wavelength λ and the frequency ν in a wave guide is given by

$$\lambda = \frac{c}{\sqrt{\nu^2 - \nu_0^2}},$$

where ν_0 is a constant. What is the group velocity of such waves?

4. (20 points) An electron in an oscillating electric field is described by the Hamiltonian operator

$$H = \frac{p^2}{2m} - (eE_0 \cos\omega t)\, x.$$

Calculate (dx/dt), (dp/dt), and (dH/dt). (*Hint:* In quantum mechanics, the time development of the expectation value of an operator A is governed by the equation

$$\frac{d}{dt}\langle A\rangle_t = \left\langle \frac{\partial A}{\partial t}\right\rangle_t + \frac{i}{\hbar}\langle[H, A]\rangle_t.)$$

5. (20 points) What is the bound state energy of a particle moving in a one-dimensional attractive delta potential

$$V(x) = -\frac{\hbar^2\lambda}{2ma}\delta(x)?$$