Useful constants and relations:

1. $hc = 1240\ \text{eV}\cdot\text{nm}$.
2. $\lambda_C = h/m_e c = 2.43\times10^{-12}$ m.
3. Fermi-Dirac distribution function $f_{FD}(E) = \dfrac{1}{Ae^{E/kT}+1}$, where A is a constant.
4. Density of states in a gas of particles: $g(E) = C\,E^{1/2}$, where C is a constant.

Problems:

1. (15 %) A particle of mass m, whose total energy is $E = 4mc^2$, collides with an identical particle at rest. If they stick together, what is the mass of the resulting composite particle?

2. (10 %) The threshold frequency of potassium is 558 nm. What is the work function for potassium? What is the stopping potential when light of 420 nm is incident on potassium?

3. (10 %) In a particular Compton scattering experiment it is found that the incident wavelength λ is shifted by 2 percent when the scattering angle $\theta = 120^\circ$. What is the value of λ?

4. (15 %) The Hamiltonian for a certain two-level system is represented by the matrix

$$H = \hbar\omega\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

If the system starts out in the state

$$\Psi(0) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$$

What is $\Psi(t)$, the state at time t.

5. The Maxwell speed distribution of a gas system is given by

$$n(v) = Cv^2 e^{-mv^2/2kT},$$

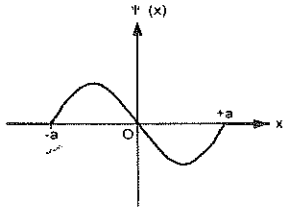
where C is a constant, m is the mass of the gas molecules, k is the Boltzmann constant, and T is the temperature of the system.

(a) (5%) Find the most probable speed of the gas.

(b) (5%) Assuming the molecules are monatomic, find the root-mean-square speed of the gas.

6. (10%) The figure below shows one of the possible energy eigenfunctions $\psi(x)$ for a particle

bouncing freely back and forth along the x-axis between impenetrable walls located at $x = -a$ and $x = +a$. The potential energy equals zero for $|x| < a$. If the energy of the particle is 2 electron volts when it is in the quantum state associated with this eigenfunction, what is its energy when it is in the quantum state of lowest possible energy? To get any credit, you must show your work, i.e., write down your equations and calculation steps.



7. (10%) The potential energy of the two atoms in a diatomic molecule as a function of the atom-atom distance, r, can be expressed as

$$U(r) = C[(e^{(r_0-r)/a} - 1)^2 - 1]$$

where C, r_0, and a are positive constants with $r_0 >> a$. Find the bond length and the binding energy in terms of C, r_0, and a.

8. (a) (5%) For silicon (Si, Z = 14) write down the appropriate electron configuration. Using the Pauli principle derive the allowed electronic states for the 4 outermost electrons. Express your answer in terms of spectroscopic notation.
   (b) (5%) What fraction of the electrons in a good conductor has energies between 0.95 E_F and E_F at $T = 0$? E_F is the Fermi energy of the conductor.
   (c) (5%) The energy gap is 0.72 eV for Ge. At what wavelengths will Ge be transparent to radiation? At what wavelengths will it begins to absorb significantly?
   (d) (5%) Why does an atom generally absorb radiation only from the ground state, while a molecule can absorb from many excited rotational or vibrational states?