Useful formula and relations:

1. Energy and momentum $E^2 = p^2c^2 + m^2c^4, E = mc^2 / \sqrt{1-\frac{v^2}{c^2}}, p = mv / \sqrt{1-\frac{v^2}{c^2}}$
2. Stefan-Boltzmann law: $R = \sigma T^4$, where $\sigma = 5.67\times10^{-8}$ W/m^2-K^4.
3. Fermi-Dirac distribution function $f_{FD}(E) = \frac{1}{Ae^{E/kT}+1}$, where A is a constant.
4. Some useful constants: $hc = 200$ eV·nm; $m_e c^2 = 0.5$ MeV, m_e is the mass of an electron.

Problems:

(1) (15 points) A particle (mass m_A) at rest decays into B particle (mass m_B) and C particle (massless). Find the energy of the outgoing B particle, in terms of the two masses, m_A and m_B.

(2) (15 points) Assume an electron is bound to a neutron by the gravitational force to form an ion. Derive an expression for the allowed electron energies based on Bohr's model.

(3) (a) (5 points) The energy reaching Earth from the Sun at the top of the atmosphere is 1.36×10^3 W/m^2, called the solar constant. Assuming that Earth radiates like a blackbody at uniform temperature, what do you conclude is the equilibrium temperature of the Earth?

(b) (5 points) What is the photoelectric effect? What does it tell us?

(c) (5 points) What is the Compton effect? What does it tell us?

(d) (5 points) Based on the theory of relativity, derive the speed of a massless particle.

(4) A particle is placed in a one-dimensional infinite potential well with a wave function $\psi(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi}{a}x)$, where $0 < x < a$. a is the width of the potential well. If this particle is in the first excited state, find

(a) (2 points) The value of n,

(b) (3 points) The probability density of the particle as a function of x,

(c) (5 points) The most probable positions of the particle.

(5) (a) (5 points) For carbon (C, Z = 6) write down the appropriate electron configuration. Using the Pauli principle derive the allowed electronic states for the 4 outermost electrons. Express your answer in terms of spectroscopic notation.

(b) (5 points) In a Stern-Gerlach experiment hydrogen atoms are used. What determines the number of lines one sees? What features of the apparatus determine the magnitude of the separation between the lines?

(6)(a) (5 points) Consider an atom with a magnetic moment μ and a total spin of 1/2. The atom is placed in a uniform magnetic field of magnitude B at temperature T. Assume that Maxwell-Boltzmann statistics is valid at this temperature, what is the ratio of atoms with spins aligned with the field to those aligned opposite the field? Express your answer in terms of the above symbols and k, the Boltzmann constant.

(b) (5 points) What fraction of the electrons in a good conductor have energies between 0.9 E_F and E_F at $T = 0$? E_F is the Fermi energy of the conductor.

(7) A particle is described by the wavefunction

$$\psi(x) = \begin{cases} A\cos(2\pi x/L) & \text{for } -L/2 \le x \le L/2 \\ 0 & \text{otherwise} \end{cases}.$$

(a) (5 points) Determine the normalization constant A.

(b) (5 points) What is the probability that the particle will be found between $x = 0$ and $x = L/4$ if a measurement of its position is made?

(8) (a) (5 points) If orbital angular momentum $\vec{L}$ is measured along the z axis to obtain a value for L_z, show that $(L_x^2 + L_y^2)^{1/2} = [\ell(\ell+1) - m_\ell^2]^{1/2}\hbar$. ℓ and m_ℓ are respectively the quantum numbers for the orbital angular momentum and the z component of the orbital angular momentum. $\hbar$ is the Planck constant h divided by 2π.

(b) (5 points) Use the uncertainty principle to make an estimate for the minimum kinetic energy (in eV) of an electron confined in a region of width 0.1 nm.