國立中正大學九十九學年度碩士班招生考試試題

系所別：物理學系　　科目：應用數學

(1) Solve the following differential equations.

(a) (10%)

$$\frac{dy}{dx}+y=e^x, \quad y(0)=1.$$

(b) (10%)

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=2x, \quad y(0)=0, \quad y'(0)=1.$$

(2) (15 %) Do the integral by contour integration,

$$\int_{-\infty}^{\infty}\frac{\cos x}{1+x^2}dx$$

(3) (15 %) Find a function $y(x)$ that minimizes the functional $I[y]$,

$$I[y]=\int_1^2\left(x^2\left(\frac{dy}{dx}\right)^2+2y^2\right)dx, \quad y(1)=2, \quad y(2)=4.$$

(4) For a real matrix A suppose we have

$$A|u_n\rangle = \lambda_n |v_n\rangle \qquad \text{and} \qquad A^T|v_n\rangle = \lambda_n |u_n\rangle ,$$

where A^T is the transpose of A and $n = 1, 2, 3\ldots$

(a) (10%) Find the eigenvalues and eigenvectors for the matrix AA^T.

(b) (10%) Show that A can be expressed as

$$A = \sum_n \lambda_n |v_n\rangle\langle u_n| .$$

(5) (15%) If the vector field $\boldsymbol{A}$ satisfies

$$\nabla \times \boldsymbol{A} = \boldsymbol{B} .$$

Rewrite the following expression in terms of $\boldsymbol{B}$

$$\left(\frac{\hbar\nabla}{i} - e\boldsymbol{A}\right) \times \left(\frac{\hbar\nabla}{i} - e\boldsymbol{A}\right)\psi ,$$

where $\hbar, e$ are real constants and ψ is a scalar function.

(6) (15%) If the function $V(x,y)$ defined in the region $x \geq 0$ and $0 \leq y \leq L$ can be expressed as

$$V(x, y) = \sum_{n=1}^{\infty} c_n \exp\left(-i\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right) .$$

Suppose at $x=L$ we have $V=V_0$ a constant, determine c_n for all values of n.