1.(15%) Given a 3×3 matrix A by

$$A = \begin{pmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{pmatrix},$$

find the inverse matrix A^{-1}.

2.(15%) Consider the first-order differential equation

$$(4x + 3y^2)\,dx + 2xy\,dy = 0\,.$$

(a) Show that this equation is not exact.
(b) Find an integrating factor of the form x^n, where n is a positive integer.
(c) Multiply the given equation by the integrating factor found in step (b) and solve the resulting exact equation.

3.(20%) Consider the second-order differential equation

$$y'' + y = \tan x\,.$$

The complementary function is defined by $y_c(x) = c_1 \sin x + c_2 \cos x$, where c_1 and c_2 are two constants. We assume that the particular integral of the differential equation be of the form

$$y_p(x) = v_1(x)\sin x + v_2(x)\cos x,$$

with the two undetermined functions $v_1(x)$ and $v_2(x)$ constrained by the additional condition

$$v_1'(x)\sin x + v_2'(x)\cos x = 0\,.$$

Find the functions $v_1(x)$, $v_2(x)$ and the particular integral $y_p(x)$ of the differential equation.

4. (10%) Suppose $g(x-y) = \dfrac{1}{2\pi}\displaystyle\sum_{n=-\infty}^{\infty} e^{in(x-y)}$, find $\displaystyle\int_{-\pi}^{\pi} f(x)\,g(x-y)\,dx$=?

5. (10%) Consider the vector field

$$\mathbf{V} = \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{(x^2+y^2+z^2)^{3/2}}\,.$$

Find the surface integral $\oint_S \mathbf{V}\cdot d\mathbf{S}$, where S is the ellipsoid

$$\frac{x^2}{4^2} + \frac{y^2}{5^2} + \frac{z^2}{6^2} = 1\,.$$

6. Evaluate the following integrals:

(a) (10%) the principal value of $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{x^2-1}\,dx$.

(b) (10%) $\displaystyle\int_0^{\infty} \frac{(\ln x)^2}{1+x^2}\,dx$.

(c) (10%) $\displaystyle\int_{-\infty}^{\infty} \frac{\exp(x/2)}{1+\cosh x}\,dx$.