1. (10 points) Given a vector field $\mathbf{V}$ that satisfies

$$\nabla \times (\nabla \times \mathbf{V}) - k^2\mathbf{V} = 0\,,$$

where k is a constant. Evaluate the surface integral $\iint_S \mathbf{V} \cdot d\mathbf{a}$, where S is an arbitrary closed surface.

2. (10 points) In cylindrical coordinates, the vector field $\mathbf{V} = \hat{\varphi}V(\rho)$. Find $(\mathbf{V} \cdot \nabla)\mathbf{V}$=?

3. Evaluate the following integrals:

   (a) (10 points) $\oint_C \frac{dz}{(z-1.01)}$, where C is the unit circle $|z| = 1$.

   (b) (10 points) $\int_0^{2\pi} \frac{d\theta}{(1-2t\cos\theta+t^2)^2}$, where $|t| > 1$.

   (c) (10 points) $\int_{-\infty}^{\infty} \frac{\sin x}{x} e^{ikx} dx$, where $|k| < 1$.

4. (15 points) Let A be a 2×2 matrix:

$$A = \begin{pmatrix} 4/3 & \sqrt{2}/3 \\ \sqrt{2}/3 & 5/3 \end{pmatrix}$$

Find the trace of the matrix A^{100}.

5. (15 points) Given a basis of R^3 as $\alpha_1 = [1,1,0]$, $\alpha_2 = [2,1,0]$, and $\alpha_3 = [1,1,1]$. Please construct an ortho-normal basis from $\{\alpha_1, \alpha_2, \alpha_3\}$ for R^3.

6. Given a differential equation of the form

$$(1-x^2)y'' - xy' + k^2y = 0$$

where k is a constant.
(a)(10 points) Determine two linear independent solutions in powers of x for $|x| < 1$.
(b)(10 points) Show that if k is a nonnegative integer n, then there is a polynomial solution of degree n.