1.(15 points) Find the work done by moving a particle in the force field $\vec{F} = 2xy\vec{i} + x^2\vec{j}$ along (a) the straight line from $(0,0)$ to $(2,1)$.(b) Can this force be derived from a scalar potential ?

2.(20 points) Solve the following differential equations :
$(a)\frac{dy}{dx} + y = e^x$, with the boundary condition $y(0) = 1$.
$(b)\frac{d^2y}{dx^2} + 4y = 3x\cos x$.

3.(15 points) (a) Find the Fourier series representation of

$$f(x) = \begin{cases} 0, -\pi < x \le 0 \\ x, 0 \le x < \pi. \end{cases}$$

(b) From your Fourier expansion show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots.$$

4. (10 points) matrix A has the form

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{pmatrix}.$$

Calculate A^{15}.

5. (15 points) Consider the following complex function

$$f(z) = \frac{1}{(z-2)(4-z)},$$

which is defined for $2 < |z| < 4$.

(a) (10 points) Find the Laurent series expansion in power of z in that region $2 < |z| < 4$. Write down the coefficients of z^n for $n = -3, -2, -1, 0, 1, 2, 3$.

(b) (5 points) In part (a), the negative powers of z does not terminate. Does this mean that $f(z)$ has an essential singularity at $z = 0$?

6. (15 points) Consider the complex function

$$f(z) = \frac{z^2 - z + 4}{z^4 + 10z^2 + 9}.$$

(a) (8 points) Find the location of the poles and the residues at each pole.

(b) (7 points) Evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2 - x + 4}{x^4 + 10x^2 + 9} dx$.

7. (10 points)Consider the following matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

where a, b, c, and d are real numbers. Can you always find a nonsingular matrix U such that $U^{-1}AU = D$ where D is diagonal? If yes, find the matrices P and D. If not, give an example that the matrix can't be diagonalized through a nonsingular matrix U.