國立中正大學九十一學年度碩士班招生考試試題

系所別：物理學系　　　　　　科　目：應用數學

共一頁

1. (10%) Use the Levi-Civita symbol ε_{ijk} to show the vector identity

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})\,.$$

2. (10%) A tensor is called reducible if it can be decomposed into parts of lower tensor rank. Find the irreducible decomposition for a general second-rank tensor A_{ij}.

3. (15%) Find the inverse matrix M^{-1}, given

$$M = \begin{pmatrix} \cos\phi & -\cos\theta\sin\phi & \sin\theta\sin\phi \\ \sin\phi & \cos\theta\cos\phi & -\sin\theta\cos\phi \\ 0 & \sin\theta & \cos\theta \end{pmatrix}\,.$$

4. (15%) Express the three-dimensional unit ~~vecor~~ vector

$$\hat{n} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

in terms of the complex variables $Z = \tan\frac{\theta}{2}e^{i\phi}$ and $\bar{Z} = \tan\frac{\theta}{2}e^{-i\phi}$.

5. (14 points) Solve the differential equation:

(a) (7 points) $(x+2)\sin y\,dx + x\cos y\,dy = 0$

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(b) (7 points) $t^2\frac{dy(t)}{dt} + 2ty(t) - y^3(t) = 0$

6. (16 points) Evaluate the following integrals.

(a) (8 points) $\int_0^{\pi} \sin^{10}\theta\, d\theta.$

(b) (8 points) $\int_0^1 \frac{dx}{\sqrt{x-x^2}(x+10)}.$

7. (20 points) Consider a 2nd order diiferential equation

$$x\frac{d^2y(x)}{dx^2} + \frac{dy(x)}{dx} + xy(x) = 0 ,$$

with initial condition $y(0) = 1$ and $y'(0) = 0$.

(a) (10 points) Find the Laplace transform of the differential equation. *i.e.*, the differential equation of the Laplace transform of $y(x)$, which is defined as

$$Y(s) = \int_0^{\infty} e^{-sx} y(x)dx .$$

Hint: you should get a first order differential equation of $Y(s)$.

(b) (10 points) Solve the differential equation which you find in part (a).