1. (10 points)

(a) It is impossible to find a vector $\vec{A}$ which satisfies the equation

$$\vec{\nabla} \times \vec{A} = \frac{\vec{r}}{r},$$

Why? ($\vec{r}$ is the position vector.

(b) It is impossible to find a scalar ψ which satisfies the equation

$$\vec{\nabla}\psi = \vec{r} \times \vec{c},$$

why? ($\vec{c}$ is a constant vector and $\vec{c} \neq 0$)

2. (20 points) Evaluate the following integrals.

(a) $$\int_0^{2\pi} \frac{\cos(3\theta)}{5 - 4\cos\theta}\, d\theta .$$

(b) $$\int_0^{\infty} \frac{\sin x}{x}\, dx .$$

3. (20 points)

(a) From the generating function of the Legendre function

$$\frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n ,$$

to show that

$$P_{n+1} = \frac{2n+1}{n+1} x P_n(x) - \frac{n}{n+1} P_{n-1}(x) .$$

(b) Given that $P_0(x) = 1$ and $P_1(x) = x$, find $P_2(x) = ?$ and $P_3(x) = ?$

4. (20 points) Let $\lambda_i (i = 1, 2, 3)$ be the eigenvalues of the matrix

$$H = \begin{pmatrix} 2 & -1 & -3 \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{pmatrix}$$

Calculate the sums (a) $\sum \lambda_i$ and (b) $\sum \lambda_i^2$.

5. (15 points) Solve the equation

$$\frac{d^2y}{dx^2} + 4y = 3x\cos x$$

6. (15 points) Using the Fourier series for the function

$$f(x) = x^4, -\pi < x < \pi$$

show that

$$\sum \frac{1}{n^4} = \frac{\pi^4}{90}$$

共一頁