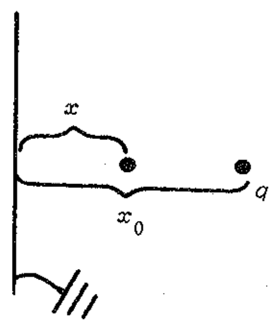
1. (20%)A particle with electric charge q is released (from rest) at the distance x_0 from the surface of a large, grounded, conducting plate. The particle is attracted by the plate, and moves toward it. What is the kinetic energy of the particle as a function of its distance x from the plate? (Neglect any energy loss by radiation.) Do you see anything unphysical about your answer?

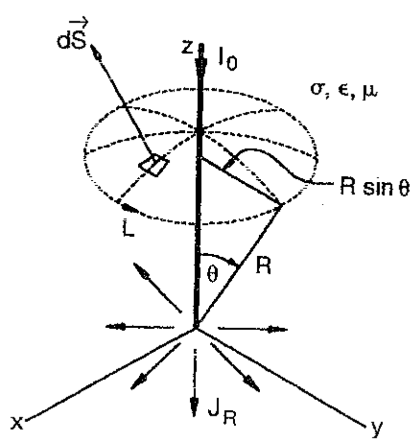


2. (30%)Consider the field surrounding a long insulated conducting filament lying along the upper half of the z axis, from $z = 0$ to $z = \infty$, as shown in the figure. The filament carriers current I_0 in the negative z direction. The medium surrounding the filamentary current and extending over all of space has conductivity σ permittivity ϵ. The tip of the filament is a spherically symmetric point source of current. Since the conducting medium extends outward in all directions, there is no more reason for current to flow in one direction than in another.

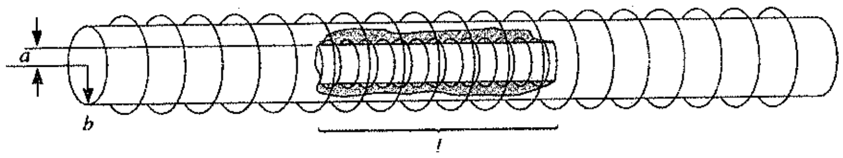
   (a) Calculate the current density $\vec{J}(R,\theta,\phi) = \vec{a}_R J_R(R)$, where $\vec{a}_R$ is the unit vector along the radial direction.

   (b) Find out the value of charges, q, at the origin (on the tip of the filament).

   (c) As shown in the figure, calculate the magnetic field $\vec{H} = \vec{a}_\phi H_\phi(R,\theta)$ by using the Ampère law, where $\vec{a}_\phi$ is the unit vector along the azimuthal direction.



3.(20%) A short solenoid (length l and radius a, with n_1 turns per unit length) lies on the axis of a very long solenoid (radius b, n_2 turns per unit length). Current I flows in the short solenoid. (a) What is the flux through the long solenoid? (b) What is the mutual inductance?



4.(10%) Derive the continuity equation from Maxwell's equations.

5.(20%) Suppose

$$\vec{E}(r,\theta,\phi,t) = A\frac{sin\theta}{r}[cos(kr-\omega t) - (1/kr)sin(kr-\omega t)]\hat{\phi}, \text{ with } \frac{\omega}{k} = c .$$

(That is, incidentally, the simplest possible spherical wave.) (a) Find the associated magnetic field. (b) Calculate the poynting vector.
(Hint: $\nabla\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r sin\theta}\frac{\partial}{\partial\theta}(sin\theta v_\theta) + \frac{1}{r sin\theta}\frac{\partial v_\phi}{\partial\phi}$ and
$\nabla\times\vec{v} = \frac{1}{r sin\theta}[\frac{\partial}{\partial\theta}(sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}]\hat{r} + \frac{1}{r}[\frac{1}{sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)]\hat{\theta} + \frac{1}{r}[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}]\hat{\phi}$)