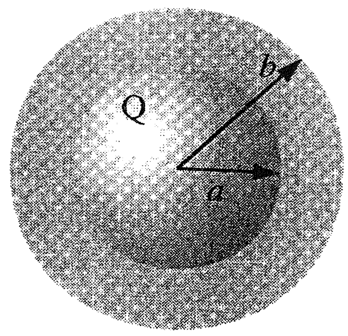
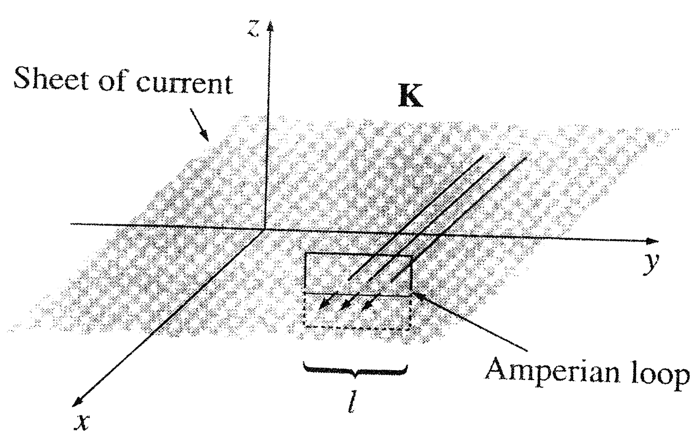
1. A metal sphere of radius a carries a charge Q. It is surrounded, out to radius b by linear dielectric material of permittivity ϵ, which is shown in the following figure. Find (a) (10%)the potential at the center (relative to infinity), and (b) (10%)the bound charge at the surface of the metal sphere.

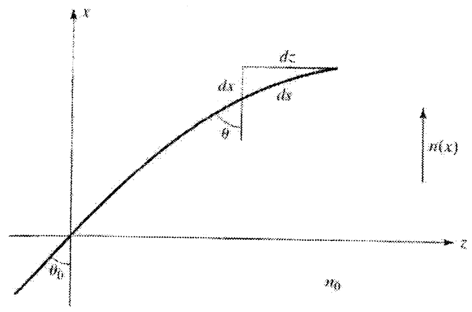


2. Two point charges $3q$ located at $(0, 0, a)$ and $-q$ located at $(0, 0, 0)$, are separated by a distance a. Find (a) (5%)the monopole moment, (b) (5%)the dipole moment, and (c) (10%)the approximate potential (in spherical coordinates) at large r (include both the monopole and dipole contributions).

3.(10%) Find the magnetic field of an infinite uniform surface current $\mathbf{K} = K\hat{x}$, flowing over the xy plane.



4. Consider a medium of continuously varying refractive index $n(x)$ as shown in the figure. Let a wave be incident from the medium of refractive index n_0 at an angle θ_0 from the vertical. To trace the path of the optical ray, let us consider a differential segment ds along the ray path, the components dx and dz in the x- and z- directions.



(a) (10%) Show that

$$\frac{dz}{dx} = \frac{n_0 \sin\theta_0}{\sqrt{n^2 - n_0^2 \sin^2\theta_0}}.$$

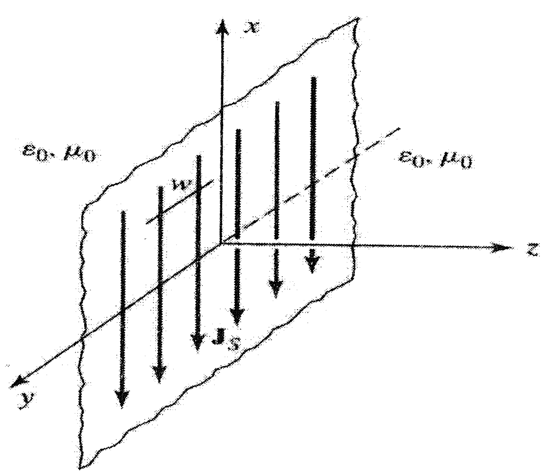
(b) (10%) If the refractive index $n(x)$ is related to x by $n^2 = n_0^2(1 - \alpha x)$, where $\alpha > 0$. Show that the ray trajectory is expressed by

$$x = \frac{\cos^2\theta_0}{\alpha} - \left(\frac{\cos\theta_0}{\sqrt{\alpha}} - \frac{\sqrt{\alpha}}{2\sin\theta_0} z\right)^2.$$

5. Consider the EM fields produced by a source consisting of an infinite sheet lying in the xy plane with the current flowing in the negative x-direction, as given by

$$\mathbf{J}_S = -J_S(t)\mathbf{a}_x \quad \text{for} \quad z = 0,$$

where $J_S(t)$ is a given function of time.



(a) (5%)From the Maxwell's equations, show that the equations for E_x, E_y, H_x, and H_y fields are expressed by

$$-\frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}, \qquad -\frac{\partial H_y}{\partial z} = J_x + \frac{\partial D_x}{\partial t},$$

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}, \qquad \frac{\partial H_x}{\partial z} = \frac{\partial D_y}{\partial t},$$

$$0 = -\frac{\partial B_z}{\partial t}, \qquad 0 = \frac{\partial D_z}{\partial t}.$$

(b) (5%)From part(a), deduce that the wave equation for E_x can be written as

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0\varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad \text{for the region} \quad z \neq 0,$$

(c) (10%)Obtain the solutions of wave equations in part(b) for E_x given by

$$E_x(z,t) = Af\left(t - \frac{z}{v_p}\right) + Bg\left(t + \frac{z}{v_p}\right),$$

and the H_y field as

$$H_y(z,t) = \frac{1}{\eta_0}\left[Af\left(t - \frac{z}{v_p}\right) - Bg\left(t + \frac{z}{v_p}\right)\right],$$

where $v_p = 1/\sqrt{\mu_0\varepsilon_0}$ and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$. Here f and g are two arbitrary functions, and A and B are two constants.

(d) (5%)By imposing a suitable boundary condition of electric fields on the sheet, show that

$$\mathbf{E}(z,t) = F\left(t \mp \frac{z}{v_p}\right)\mathbf{a}_x \quad \text{for} \quad z \gtrless 0,$$

and

$$\mathbf{H}(z,t) = \pm\frac{1}{\eta_0}F\left(t \mp \frac{z}{v_p}\right)\mathbf{a}_y \quad \text{for} \quad z \gtrless 0,$$

where $Af(t) = Bg(t) = F(t)$.

(e) (5%)By imposing a suitable boundary condition of magnetic fields on the sheet, deduce the electric and magnetic fields obtained as:

$$\mathbf{E}(z,t) = \frac{\eta_0}{2}J_S\left(t \mp \frac{z}{v_p}\right)\mathbf{a}_x \quad \text{for} \quad z \gtrless 0,$$

and

$$\mathbf{H}(z,t) = \pm\frac{1}{2}J_S\left(t \mp \frac{z}{v_p}\right)\mathbf{a}_y \quad \text{for} \quad z \gtrless 0.$$