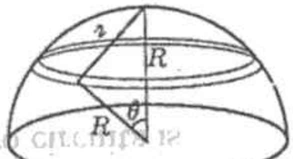
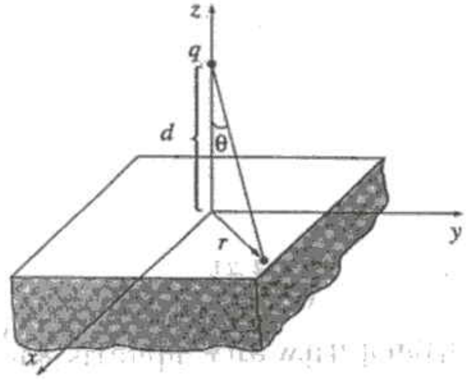
1.(10%) Find the general solution V to Laplace's equation in spherical coordinates $(r, \theta\, \phi)$, for the case where V depends only on r. Do the same for cylindrical coordinates (s, ϕ, z), assuming V depends only on s.

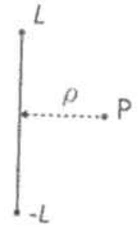
2.(20%) An inverted hemispherical bowl of radius R, which is shown in the following figure, carries a uniform surface charge density σ. Find the potential difference between the "north pole" and the center.



3.(20%) Suppose the entire region below the plane $z = 0$ is filled with uniform linear dielectric material of susceptibility χ_e. A point charge q is situated a distance d above the origin which is shown in the following figure. (a) Calculate the total bound charge. (b) Calculate the force on charge q.



4. (20%)Calculate the magnetic field $\vec{B}$ for a long straight wire with length $2L$ and current I, via the vector potential, $\vec{A}$. (Note: $d\vec{A} = \frac{\mu_0}{4\pi}\frac{I}{r}dl$)



(a) What is the vector potential $\vec{A}$ at the point P with a distance ρ from the middle of the wire?

(b) Calculate the magnetic field $\vec{B}$ in the cylindrical coordinate.
Hint:
$$\nabla \times \vec{A} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{z}\frac{1}{\rho}\left(\frac{1}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\right)$$

(c) In the limiting case of $\rho \ll L$, what is the expression for $\vec{A}$? What is the magnetic field for $\rho \ll L$?

5. (10%) Show that the mutual inductance between the two circuits is

$$M_{ab} = \frac{\mu_0}{4\pi}\oint_a \oint_b \frac{d\vec{l_a} \cdot d\vec{l_b}}{r},$$

where $d\vec{l_a}$ and $d\vec{l_b}$ are vector line elements on the circuit a and b, respectively.

6. (20%) Derive the wave equations for the electric and magnetic fields, ($\vec{E}$ & $\vec{H}$) in the conducting media with the conductivity σ.