1. (15%)

One of the following two expressions of electric field $\vec{E}$ is an impossible electrostatic field. Which one?

(a). $\vec{E} = xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}$ ;

(b). $\vec{E} = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$.

For the possible one, find the potential $V(x, y, z)$, using the origin as the reference point.

2. (15%)

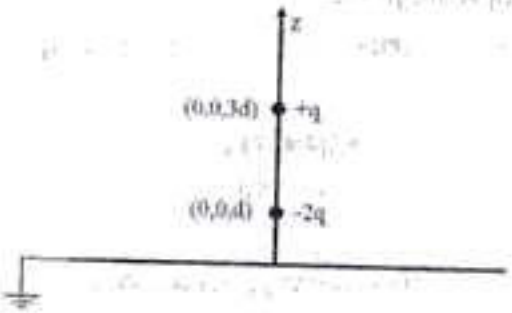
One infinite grounded conducting plane is located on $z = 0$ plane. In the region $z > 0$, we put two point charges $+q$ and $-2q$ with Cartesian coordinates $(0, 0, 3d)$ and $(0, 0, d)$, respectively.

(a). Write down equations for the potential $V(x, y, z)$ for $x > 0, y > 0,$ and $z > 0$.

Write down the boundary conditions for solving these equations.

(b). State the uniqueness theorem and why the above problem can be solved by the method of images.

(c ). Find the force on the charge $+q$.




3. (20%)

A point charge $q$ is imbedded at the center of a sphere of linear dielectric material (with susceptibility $\chi_e$ and radius $R$). Find the electric field, the polarization, and the bound charge densities (volume charge density $\rho_b$ and surface charge density $\sigma_b$). What is the total bound charge on the surface? Where is the compensating negative bound charge located?

4. (15%)

Show that bound volume and surface currents are $\mathbf{J}_b = \nabla \times \mathbf{M}$ and $\mathbf{K} = \mathbf{M} \times \hat{\mathbf{n}}$ ($\hat{\mathbf{n}}$ is a normal unit vector) for a magnetized object with the magnetization $\mathbf{M}$.

5. (15%)

(a) Write down the Maxwell's equations.

(b) Derive the equations for the vector potential $\mathbf{A}$ and the scalar potential $\phi$ in solving the Maxwell's equations?

(c) In the Lorentz gauge, what are the simplified equations derived in part (b)? Discuss some properties of the solutions of the equations.

6. (20%)

(a) What is the wave equation of the electric field propagating in a source-free dispersive medium with a complex permittivity $\tilde{\varepsilon}$.

(b) The permittivity of the medium obtained from a classic model can be expressed as

$\frac{\tilde{\varepsilon}}{\varepsilon_0} = 1 + \frac{Nq^2}{m\varepsilon_0} \sum_{j=1} \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j \omega}$, where $m$ and $q$ are the mass and charge of an electron, respectively. Here $N$ is the number of molecules per unit volume. There are $f_j$ electrons with natural frequency $\omega_j$ and damping strength $\gamma_j$ in each molecule. Calculate the absorption coefficient and the index of refraction.

(c) What is the group velocity of a wave propagating in this dispersive medium with vanishing damping strengths ($\gamma_j = 0$, for all $j$)?