1. (10%) Show that $\nabla \times (f\vec{A}) = (\nabla f) \times \vec{A} + f(\nabla \times \vec{A})$, where f is a scalar function and $\vec{A}$ is a vector function.

2. (20%) Find the voltage drop across each dielectric in Fig 1, where $\varepsilon_{r1}=2.0$ and $\varepsilon_{r2}=5.0$. The inner conductor is at $r_1=2$ cm and the outer at $r_2=2.5$cm, with the dielectric interface halfway between.
$(\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N\cdot m^2)$

3. (20%) A long straight wire of radius a has a circular hole of radius b parallel to the axis of the wire but displaced from the center by a distance c as shown in Fig 2. A current I flows in the wire and is uniformly distributed across the conductor. Find the magnetic field everywhere in space.

Fig 1

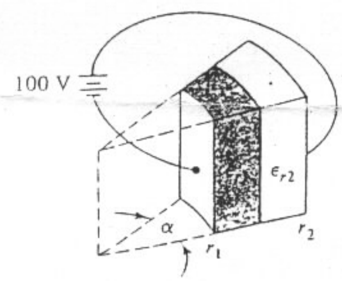
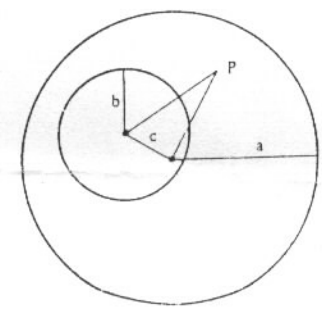


Fig2



b + c < a

4. (10%) Write down the Maxwell equations for the region with the current **J** and charge density ρ.

5.(20%) Consider in vacuum an electromagnetic plane wave propagating along the x-axis with the wave vector k and the electric wave amplitude E_0.
(a) What is the Poynting vector?
(b) What is the energy density and momentum density of the wave?
(c) What are the time -averages for three of them?

6.(20%) Assume that hall conductivity tensors of some system are $\sigma_{yy} = \sigma_{xx} = 0$, and $\sigma_{xy} = -\sigma_{yx} \neq 0$.
Let $\Phi(t)$ is the time-dependent magnetic flux inside a loop Γ in $x-y$ plane.

(a)(7%) Show that the field at the perimeter induces a current **J** which obeys:

$$\hat{z} \bullet \mathbf{J} \times d\mathbf{r} = \sigma_{xy}\mathbf{E} \bullet d\mathbf{r}$$

(b) (13%)With the result in (a), show that

$$\frac{dQ}{dt} = \sigma_{xy}\frac{d\Phi}{dt}$$

where Q is the total charge inside Γ.