1. (15 %) Construct a phase diagram for the potential $U(x) = -\frac{1}{3}x^3$.

2. (15 %) A pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k, Find Lagrange's equations of motion. (You don't need to solve the equations)

3. (20 %) A body of rest mass m, traveling initially at a speed 0.8 c, makes a completely inelastic collision with an identical body initially at rest. (a) What is the speed of the resulting single body ? (b) What is its rest mass ?

4.(20%) Calculate the centrifugal acceleration, due to Earth's rotation, of a particle on the surface at the equator. Compare this result with the gravitational acceleration. Compute also the centrifugal acceleration due to the motion of Earth about the sun and justify that this acceleration may be neglected compared with the acceleration caused by the Earth axial rotation.

Note that the gravitational acceleration is 980 cm/sec^2, the radius of the Earth is 6.4×10^8 cm, and the distance between the centers of sun and Earth is 1.5×10^{13} cm. The effective force on the surface of Earth is written as

$$\vec{F}_{\rm eff} = m\vec{g}_0 - m\vec{\omega}\times[\vec{\omega}\times\vec{r}'] - 2m\vec{\omega}\times\vec{v}_r.$$

5.(20%) Give a brief discussion on the following topics:
(a) Open, closed, and almost closed orbits in a central potential $U(r)$.
(b) "Weak" form and "strong" form of the Newton's Third Law.
(c) The coefficient of restitution (ϵ); Newton's rule on the inelasticity of two body collision.
(d) The feedback effect of the coupled oscillations.

6.(10%) Find the principal moments of inertia of the following moment-of-inertia tensor

$$\{I\} = \begin{pmatrix} \frac{2}{3}\beta & -\frac{1}{4}\beta & -\frac{1}{4}\beta \\ -\frac{1}{4}\beta & \frac{2}{3}\beta & -\frac{1}{4}\beta \\ -\frac{1}{4}\beta & -\frac{1}{4}\beta & \frac{2}{3}\beta \end{pmatrix}.$$