1. (15 %)

Starting from rest at the top, a small sphere of radius r rolls without slipping off a large fixed sphere of radius R. At what angle (with respect to the vertical) will the small sphere leave the surface of the big sphere?

2. (15%)

Compute the gravitational self-energy (energy of assembly piecewise from infinity) of a uniform sphere of mass M and radius R.

3. (20%)

A simple pendulum of length b and bob with mass m is attached to a massless support moving vertically upward with constant acceleration a. Determine the equations of motion and find the period for small oscillations.

4. (20%)

A homogeneous solid ball of radius R and mass M has the moment of inertia I with respect to the axis passing through the center of the sphere. If one removes a concentric solid sphere of radius r ($r < R$) from the ball and let the moment of inertia of the punctured ball be I'. Determine the ratio I/I'.

5. (15%)

Suppose a particle of mass m has equation of motion $m\ddot{\mathbf{r}} = \frac{\alpha}{r^3}\mathbf{r} \times \ddot{\mathbf{r}}$, where $\mathbf{r}$ is the position vector of the particle, $r = |\mathbf{r}|$, and α is a constant. Please show that $T \equiv \frac{1}{2}m\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}$ and $J \equiv m\mathbf{r} \times \dot{\mathbf{r}} + \frac{\alpha}{|\mathbf{r}|}\mathbf{r}$ are constants of motion.

6. (15%)

Consider a charged particle of unit mass moves in two dimensions under the influence of dipole. The Lagrangian is given by

$$L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{\lambda \cos\theta}{r^2}$$

where r, θ are the plane polar coordinates of the charged particle, and λ is the constant strength of the dipole. Please derive the generalized momenta p_r and p_θ conjugate to r and θ, and write down the Hamiltonian.

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