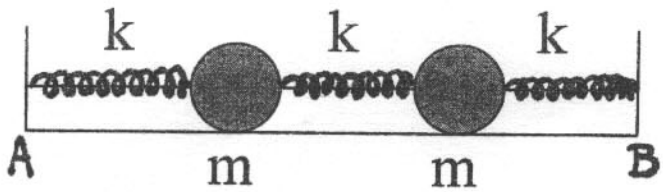
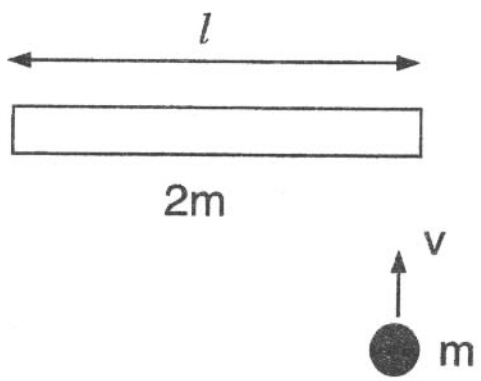
1. (a) (10 %) If a rigid body rotates about a fixed axis with angular velocity ω, prove that the kinetic energy of rotation is $T=1/2\ I\ \omega^2$ where I is the moment of inertia about the axis. (b) (15 %) Find the moment of inertia of a right circular cone of height h and radius a about its axis.

2. (25 %)Two equal masses m are connected by springs having equal spring constant k, so that the masses are free to slide on a frictionless table AB. The walls at A and B to which the ends of the springs are attached are fixed. (a) set up the equations of motion of the masses, (b) find the normal frequencies, and (c) find the normal modes of vibration for the system



3. (25 %) A uniform rod of length l and mass $2m$ rests on a smooth horizontal table. A point mass m moving horizontally at right angles to the rod with an initial velocity v collides with one end of the rod and sticks to it. Determine (a) the center of mass, the moment of inertia, and the angular velocity of the system after the collision, (b) the position of the point on the rod which remains stationary immediately after the collision, and (c) the change in kinetic energy of the system as a whole as a result of the collision.



4. (25 %) Derive the equation of motion of a particle of mass m subject to restoring and frictional forces of magnitude kx and bdx/dt respectively, where x is its displacement and k and b are positive constants.
Show that $x = A\exp(-\gamma t)\cos(\omega t + \phi)$ is only a solution of the equation of motion for $4km > b^2$ and determine the value of γ. (where the γ, ω, A, and ϕ are real constants.) Comment on the physical meaning of this solution.