Qualify Exam.(Q.M./2008/Fall)

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1 (a) (15 points) Consider a particle of mass m moving in a one-dimensional square well

$$V(x) = \begin{cases} 0 & \text{for } |x| < L, \\ V_0 & \text{for } |x| \geq L, \end{cases}$$

where V_0 is a positive constant. For any values of V_0 and L, is there always a bound state for the particle? Justify your answer

(b) (15 points) Consider the same problem as in (a) but now in three-dimensional space: namely, if the particle moves in the three-dimensional potential

$$V(r) = \begin{cases} 0 & r < R, \\ V_0 & r \geq R, \end{cases}$$

would there always be a bound-state solution? Justify your answer.

2. Consider two spin-1/2 particles of the same mass but opposite charges that interact with each other via the Hamiltonian

$$H_1 = \alpha \vec{S}_1 \cdot \vec{S}_2 ,$$

where α is a constant and $\vec{S}_i$ are spin operators of each particle. Suppose there is an external magnetic field $\vec{B}$ that induces an additional term in the Hamiltonian (ignore orbital motions)

$$H_2 = \beta \vec{B} \cdot \left(\vec{S}_1 - \vec{S}_2 \right) .$$

(a) (10 points) Find the eigenenergy and eigenstates of the total system The Pauli matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(b) (10 points) Suppose at $t = 0$ the system is in the state $|S_{1z} = \uparrow, S_{2z} = \downarrow\rangle$, find the state of the system at time $t > 0$

3. (a) (10 points) Let E_n and $|E_n\rangle$ be eigenvalues and eigenstates of a Hamiltonian H In particular, E_0 is the lowest eigenvalue of H, i e , the ground-state energy. Let $|\Psi\rangle$ be a trial state Show that

$$E[\Psi] \equiv \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \geq E_0.$$

(b) (15 points) For the attractive delta function potential $V = -aV_0\delta(x)$ use a Gaussian trial function $\Psi = \exp(-\lambda x^2)$ to compute the upper bound on E_0.

4. Using the Heisenberg uncertainty relation $\Delta P_x \Delta X \geq \hbar/2$, etc. to estimate the ground state energy of the following systems.

(a) (10 points) 1-D simple harmonic oscillator:

$$H = \frac{P^2}{2m} + \frac{m\omega^2 X^2}{2}$$

(b) (15 points) the Hydrogen atom:

$$H = \frac{P_x^2 + P_y^2 + P_z^2}{2m} + \frac{e^2}{(X^2 + Y^2 + Z^2)^{1/2}}$$