

九十六學年度

國立中正大學

博士班研究生資格考試

所別：物理研究所

科目：量子力學

(※題目共一張六大題)

96.09.12

1 (15%) Let $x(t)$ be the coordinate operator for a free particle in one dimension in the Heisenberg picture. Evaluate

$$[x(t), x(0)].$$

2 (15%) Consider a spin 1/2 system represented by the normalized spinor $\begin{pmatrix} 1 \\ 2 \end{pmatrix} / \sqrt{5}$ what is the probability that a measurement of $S_x = \hbar\sigma_x/2$ yields the value $+\hbar/2$?

3 (20%) A system is described by the following Hamiltonian:

$$H = \left[\frac{5}{3} a^\dagger a + \frac{2}{3} (a^2 + (a^\dagger)^2) \right] \hbar\omega$$

where a and a^\dagger are the usual raising and lowering operators for harmonic oscillator. Solve for the energy spectrum of this system and the ground-state wave-function

4 (20%) The Hamiltonian of a spin-1/2 system is given by

$$H = \frac{a}{\hbar^2} S_z^2 + \frac{b}{\hbar^2} (S_x^2 - S_y^2), \quad S_i = \hbar\sigma_i/2$$

where $b \ll a$ and treat the second term as a perturbation. Calculate the unperturbed energies and the first corrections using perturbation theory.

5 (15%) A particle of spin 1 moves in a central potential of the form

$$V(r) = V_1(r) + \frac{\mathbf{S} \cdot \mathbf{L}}{\hbar^2} V_2(r) + \frac{(\mathbf{S} \cdot \mathbf{L})^2}{\hbar^4} V_3(r)$$

What are the values of $V(r)$ in the states $J = L + 1$ and $L - 1$? (J is the total angular momentum, $\mathbf{J} = \mathbf{S} + \mathbf{L}$)

6 (15%) An electron moves in the presence of a uniform magnetic field in the z -direction ($\mathbf{B} = B\hat{z}$). Evaluate

$$[\Pi_x, \Pi_y],$$

where

$$\Pi_x = p_x - \frac{eA_x}{c}, \quad \Pi_y = p_y - \frac{eA_y}{c}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$