Qualify Exam.(Q.M./2006/Fall)

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1. (20 points) For a Hermitian operator A, the eigenvalue equation is defined by

$$A|\lambda_a >= \lambda_a|\lambda_a > .$$

(a)Show that its eigenvalues λ_a are real numbers. (b) If no degeneracy occurs, show that eigenvectors are orthogonal.

2. (20 points) Compute its eigenvalues and eigenfunctions of the potential $V(x) = -aV_0\delta(x)$.

3. (20 points) The Hamiltonian of the hydrogen atom is given by

$$H = \frac{P_x^2 + P_y^2 + P_z^2}{2m} - \frac{e^2}{(X^2 + Y^2 + Z^2)^{1/2}}.$$

Using the uncertainty principle to estimate the ground state energy E_0 and the size of the ground-state wave function.

(4)

Consider an angular momentum 1 system, represented by the state vector

$$u = \frac{1}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$$

What is the probability that a measurement of L_x yields the value 0?

(5)

Consider an electron in a uniform magnetic field along the z direction. Let the result of a measurement be that the electron spin is along the positive y direction at $t = 0$. Find the Schrödinger state vector for the spin, and the average polarization (expectation value of s_x) along the x direction for $t > 0$.

(6)

Employing first order perturbation theory, calculate the energy of the first three states for an infinite square well of width a, whose portion AB has been sliced off. (Note: The line OA is a straight line).