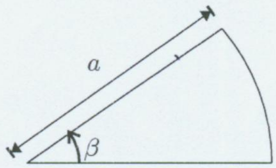
indicated in the sketch. All boundaries, $\rho = a, \phi = 0, \phi = \beta$, are grounded. Find out the potential inside this geometry.



2. (a) (10%) Show that the Green function $G(x, y; x', y')$ appropriate for Dirichlet boundary conditions for a square two-dimensional region, $0 \le x \le 1, 0 \le y \le 1$, has an expansion

$$G(x, y; x', y') = 2\sum_{n=1}^{\infty} g_n(y, y')\sin(n\pi x)\sin(n\pi x')$$

where $g_n(y, y')$ satisfies

$$\left(\frac{\partial^2}{\partial y'^2} - n^2\pi^2\right)g_n(y, y') = -4\pi\delta(y' - y) \text{ and } g_n(y, 0) = g_n(y, 1) = 0$$

(b) (10%) Taking for $g_n(y, y')$ appropriate linear combinations of $\sinh(n\pi y')$ and $\cosh(n\pi y')$ in the two regions, $y' < y$ and $y' > y$, in accord with the boundary conditions and the discontinuity in slope required by the source delta function,show that the explicit form of G is

$$G(x, y; x', y') = 8\sum_{n=1}^{\infty}\frac{1}{n\sinh(n\pi)}\sin(n\pi x)\sin(n\pi x')\sinh(n\pi y_<)\sinh[n\pi(1 - y_>)]$$

where $y_<(y_>)$ is the smaller (larger) of y and y'.

3. (a) (5%)Solve the Green function for a spherical shell bounded by $r = a$ and $r = b$ $(a < b)$, it can be expressed as

$$G(\vec{x}, \vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y^*_{lm}(\theta', \phi')Y_{lm}(\theta, \phi)}{(2l + 1)[1 - (\frac{a}{b})^{2l+1}]}\left(r^l_< - \frac{a^{2l+1}}{r^{l+1}_<}\right)\left(\frac{1}{r^{l+1}_>} - \frac{r^l_>}{b^{2l+1}}\right)$$

(b) (5%)Make using of the result in part(a) to calculate the potential inside the grounded **sphere** with a uniformly charged ring of radius a which is located on the x-y plane, and total charge Q, and b is the radius of the sphere.

(c) (5%)Calculate the potential inside a grounded **sphere**, inside which a uniformly charged rod with length $2b$ and total charge Q is placed on the z axis between the north and south poles of the sphere.

4.(15%) The Dirac delta function in three dimensions can be taken as the improper limit as $\alpha \to 0$ of the Gaussian function

$$D(\alpha; x, y, z) = (2\pi)^{-3/2}\alpha^{-3} \exp\left[-\frac{1}{2\alpha^2}(x^2 + y^2 + z^2)\right].$$

Consider a general orthogonal coordinate system specified by the surfaces $u =$ constant, $v =$ constant, $w =$ constant, with the length elements du/U, dv/V, dw/W in the three perpendicular directions. Show that

$$\delta(\vec{r} - \vec{r'}) = \delta(u - u')\delta(v - v')\delta(w - w') \cdot UVW$$

by considering the limit of the Gaussian above. Note that as $\alpha \to 0$ only the infinitesimal length element need be used for the distance between the points in the exponent.

5.(20%) (a) Two halves of a long hollow conducting cylinder of inner radius b are separated by small lengthwise gaps on each side, and are kept at different potentials V_1 and V_2. Show that the potential inside is given by

$$\Phi(\rho, \phi) = \frac{V_1 + V_2}{2} + \frac{V_1 - V_2}{\pi} \tan^{-1}\left(\frac{2b\rho}{b^2 - \rho^2}\cos\phi\right),$$

where ϕ is measured from a plane perpendicular to the plane through the gap.
(b) Calculate the surface-charge density on each half of the cylinder.

Note that the Laplacian of a scalar function ψ in the cylindrical coordinate (ρ, ϕ, z) is

$$\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}.$$

6.(15%) Give a brief explanation on the following items:
(a) The near zone and the far zone in a radiating system.
(b) The Born Approximation for the scattering amplitude.
(c) The blueness of the sky and the redness of the sunset.