1.(20%) The electric potential is given by the expression

$$\Phi = A\frac{e^{-\alpha r}}{r}$$

where A and α are constants.
(a) What is the charge density?
(b) What is the total charge Q?

2.(20%) Two point charges, $3q$ and $-q$ are separated by a distance a and are located at $(0,0,a)$ and $(0,0,0)$, respectively.
(a) Find the monopole moment and the dipole moment.
(b) Find the approximate potential (in spherical coordinates) at large r.

3.(10%) (a) Explain the physical meanings for (i) the near zone, and (ii) the far zone in a radiating system.
(b) Are the following two statements right? If not, correct these statements. (i) A charge radiates whenever it is moving in whatever manner. (ii) Radiation emitted by an antenna has angular distribution characteristic of dipole radiation when the wavelength is short compared with the antenna.

4.(25%) A line charge with linear charge density τ is placed parallel to, and a distance R away from, the axis of a conducting cylinder of radius b held at a fixed voltage such that the potential vanishes at infinity. Find
(a) the magnitude and position of the image charge(s);
(b) the potential at any point (expressed in polar coordinates with the origin at the axis of the cylinder and the direction from the origin to the line charge as the x axis), including the asymptotic form far from the cylinder;
(b) the induced surface-charge density, and plot it as a function of angle for $R/b = 2, 4$ in units of $\tau/2\pi b$;
(d) the force on the charge.

5.(25%) The Dirichlet Green function for the unbounded space between the planes at $z = 0$ and $z = L$ allows discussion of a point charge or a distribution of charge between parallel conducting planes held at zero potential.
(a) Using cylindrical coordinates show that one form of the Green function is

$$G(\mathbf{x}, \mathbf{x}') = \frac{4}{L}\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} \sin\left(\frac{n\pi z}{L}\right) \sin\left(\frac{n\pi z'}{L}\right) I_m\left(\frac{n\pi}{L}\rho_<\right) K_m\left(\frac{n\pi}{L}\rho_>\right),$$

where $I_m(x)$ and $K_m(x)$ are modified Bessel functions.
(b) Show that an alternative form of the Green function is

$$G(\mathbf{x}, \mathbf{x}') = 2\sum_{m=-\infty}^{\infty}\int_0^{\infty} dk\, e^{im(\phi-\phi')} J_m(k\rho) J_m(k\rho') \frac{\sinh(kz_<)\sinh[k(L-z_>)]}{\sinh(kL)},$$

where $J_m(x)$ is Bessel function.