國立中正大學九十九學年度學士班三年級轉學生招生考試試題

學系別：物理學系　　　　科目：應用數學

第 1 節　　　　

1. Solve each of the following initial-value problems.

(1a) (10 %) $\frac{d^2y}{dx^2}+y=0,\quad y=1 \; and \; \frac{dy}{dx}=0 \;\; for \; x=0$

(1b) (10 %) $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0,\quad y=0 \; and \; \frac{dy}{dx}=1 \; for \; x=0$

2. (15 %) Solve the differential equation

$$x\frac{dy}{dx}+(x+y)=0,\quad y=1 \; for \; x=1.$$

3. (15 %) Consider the equation

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

Given that $y(x)=x$ is one of its solutions, find a second linearly independent one.

4. For the matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

(4a)(12%) Find the **eigenvalues** and **orthonormal eigenvectors**.

(4b)(8%) Compute the determinant of $\exp(A^2)$.

5. Given the function $\phi(x,y,z) = x^3y^2z$.

(5a)(5%) Find the plane which tangent the surface $\phi(x,y,z) = 2$ at $(1,1,2)$.

(5b)(5%) Compute $\nabla \times \nabla\phi$.

6. (6a)(10%) Use the divergence theorem to evaluate the surface integral

$$I_1 = \int\int_S \mathbf{v} \cdot \hat{\mathbf{n}} dS$$

where $\mathbf{v} = (x,0,0)$, S is the open surface of the hemisphere $\{(x,y,z)|x^2+y^2+z^2 = 4, z \geq 0\}$ and $\hat{\mathbf{n}}$ is the outward-pointing normal.

(6b)(10%) Use Stokes's theorem to evaluate the line integral

$$I_2 = \oint_C \mathbf{u} \cdot d\mathbf{r}$$

where $\mathbf{u} = (2x-y, -y^3, -y^3z)$ and C is the contour $\{(x,y,z)|x^2+y^2 = 1, z = 0\}$.