國立中正大學九十九學年度學士班二年級轉學生招生考試試題

學系別：數學系、地球與環境科學系、物理學系
化學暨生物化學系、資訊工程學系
機械工程學系、通訊工程學系

科目：微積分

第 1 節

PART I. Fill in the blank (9 points each. No partial credits)

1. $\lim_{n\to\infty}(\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{2n})$ = _____.

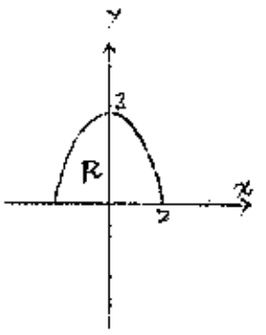
2. If $f(x)$ is a function satisfying $f''(x)=\frac{\sin x}{x}$, $f(\frac{\pi}{2})=a$, $f(\frac{7\pi}{3})=b$,
then $\int_{\pi/3}^{7\pi/3} f(x)\,dx$ = _____. (express your answer in terms of a and b).

3. If $\int_1^\infty(\frac{x}{ax^2+1}-\frac{2}{3x})dx$ is a convergent improper integral, then a=_____
and the value of the improper integral is _____.

4. The interval of convergence of the power series $\sum\frac{4^n}{n+1}(x-2)^n$ is _____.

5. The line integral $\int_C F\cdot dr$ = _____, where $F=\frac{-yj+xi}{4x^2+y^2}$,
and C is the unit circle traced counterclockwise.

6. $\iint_R \sin(9x^2+4y^2)dA$=_____, where R is part of the ellipse in the right figure.



7. $\lim_{n\to\infty}\frac{1}{n}(1+\sqrt[3]{2}+\sqrt[3]{3}+...+\sqrt[3]{n})$ = _____.

8. The area of the region that is inside both curves: the cardioid $r=1+\cos\theta$ and the circle $r=1$ is _____.

PART II. Show your work to get full credits. (14 points each)

1. (a) Show that $\int_0^\infty xe^{-nx}dx=\frac{1}{n^2}$, $n=1,2,3...$.

(b) Use above result to show that $\int_0^\infty\frac{x}{e^x-1}dx=\sum_{n=1}^\infty\frac{1}{n^2}$

2. Let $y=h(t)=\frac{1}{1+9e^{-0.08t}}$ then y satisfies the differential equation $y'=f(y)$.
(a) Find the function $f(y)$.
(b)The function $y=h(t)$ has a unique inflection point in $(0,\infty)$, find this point.