

國立中正大學九十八學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系、
學系別：化學暨生物化學系、資訊工程學系、經濟學系、科目：微積分
電機工程學系、機械工程學系、通訊工程學系

第 1 節

第 1 頁，共 1 頁

★ Write down your answers without calculations in problems 1 ~ 5.

1. Evaluate the following limits:

(a). $\lim_{x \rightarrow 1} \frac{(1 - \sqrt{x})(1 - \sqrt[3]{x})(1 - \sqrt[4]{x})}{(1 - x)^3} = \underline{\hspace{2cm}}$ (5 pts.)

(b). $\lim_{x \rightarrow 0} (e^{3x} + 2x)^{\frac{1}{x}} = \underline{\hspace{2cm}}$ (5 pts.)

(c). $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \cdots + \frac{1}{\sqrt{n^2 + n}} \right) = \underline{\hspace{2cm}}$ (5 pts.)

(d). $\lim_{n \rightarrow \infty} \frac{1}{n^{2p+1}} (1^p + 2^p + \cdots + n^p) = \underline{\hspace{2cm}}$ where $p > 0$. (5 pts.)

2. Suppose that $f(x) = \begin{cases} ax - b & \text{if } x \leq 1 \\ x^3 & \text{if } x > 1 \end{cases}$ is a differentiable function on \mathbb{R} . Then

$$\lim_{x \rightarrow 0^+} \frac{1}{x\sqrt{x}} \left(a \tan^{-1} \left(\frac{\sqrt{x}}{\sqrt{a}} \right) - b \tan^{-1} \left(\frac{\sqrt{x}}{\sqrt{b}} \right) \right) = \underline{\hspace{2cm}} \quad (15 \text{ pts.})$$

3. Let $a_n = \int_0^{\frac{\pi}{2}} (\sin x)^n dx$ where $n \in \mathbb{N} \cup \{0\}$. Then

(a). $\frac{a_n}{a_{n-2}} = \underline{\hspace{2cm}}$ where $n \in \mathbb{N}$ and $n \geq 2$. (10 pts.)

(b). $a_n = \underline{\hspace{2cm}}$ if n is even. (5 pts.)

(c). $a_n = \underline{\hspace{2cm}}$ if n is odd. (5 pts.)

4. Assume that $f(x) = \sqrt{x^3}$. Define $\Gamma = \{(x, f(x)) \in \mathbb{R}^2 | 0 \leq x \leq 4\}$ and

$$R = \{(x, y) \in \mathbb{R}^2 | 0 \leq x \leq 4, 0 \leq y \leq f(x)\}.$$
 Then

(a). The arc length of $\Gamma = \underline{\hspace{2cm}}$. (5 pts.)

(b). The volume of the solid obtained by rotating R about the x -axis is $\underline{\hspace{2cm}}$. (5 pts.)

5. Suppose that $\iint_S \left(\sqrt{\frac{y}{x}} \right)^3 + \sqrt{xy} \, dx dy = \iint_T (v^3 + u) \frac{\partial(x, y)}{\partial(u, v)} du dv = A$,

$$\text{where } S = \{(x, y) \in \mathbb{R}^2 | 1 \leq xy \leq 9, 1 \leq y/x \leq 4\}.$$
 Then

(a). $T = \underline{\hspace{2cm}}$. (5 pts.)

(b). $A = \underline{\hspace{2cm}}$. (5 pts.)

6. Show that if a_0, a_1, \dots, a_n are real numbers satisfying

$$\frac{a_0}{1} + \frac{a_1}{2} + \cdots + \frac{a_n}{n+1} = 0,$$

then the equation $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n = 0$ has at least one real root. (10 pts.)

7. Find extrema of $f(x, y, z) = x^3 + y^3 + z^3$ subject to $x^2 + 2y^2 + 3z^2 = 4$. (15 pts.)