CALCULUS

PART I (70%) - FILL IN THE BLANKS

7% each blank. NO partial credits.

(1) Let $f(x) = x^5 + 3x + 1$, then $(f^{-1})'(5) =$ _______.

(2) Let $C(x) = \int_0^x \sin^2 t\, dt$. Then $\lim_{x\to 0} C(x)/x^3 =$_______. (Answer "None" if the limit does not exist.)

(3) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{\sqrt{n^2+ni}} =$ _______. (Answer "None" if the limit does not exist.)

(4) Determine whether the integral converges. Then $\int_1^\infty \frac{(x+1)\ln x}{x^3} dx$ _______ (Answer "Converges" if the integral does exist and "Diverges" if the integral does not exist.)

(5) The convergent set (interval of convergence) for $\sum_{k=1}^{\infty} \frac{x^k}{(k+1)2^k}$ is _______.

(6) Let

$$f(x) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Then the gradient $\nabla f(0,0) =$_______ and $\lim_{(x,y)\to(0,0)} f(x,y) =$_______. (Answer "None" if the limit does not exist.)

(7) The tangent plane to $z = x^2 + y^2$ at $(1,1,2)$ is _______.

(8) $\int_0^\infty e^{-x^2} dx =$ _______.

(9) Let Ω be region in the first-quadrant bounded by $x^2+y^2=4$, $x^2+y^2=9$, $x^2-y^2=1$ and $x^2-y^2=4$. Then $\int\int_\Omega xy\,dxdy =$ _______.

PART II (30%) - COMPUTATIONAL PROBLEMS

Show all your work. NO CREDITS if only present answers.

(1) Let $f(x, y) = x^2 + y^2 - 2x - 2y + 4$ on $D = \{(x, y) : x^2 + y^2 \leq 25\}$. Find the absolute extreme values. (10 points)

(2) Find $\lim_{x\to 0} x \cdot [\frac{1}{x}]$, where the greatest integer function, $[x]$, is defined by the greatest integer less than or equal to x. (10 points)

(3) Use Green's Theorem to find the area of Ω, where $\Omega = \{(x, y) : \frac{x^2}{4} + \frac{y^2}{9} \leq 144\}$. (10 points)