

國立中正大學九十六學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系、
學系別：化學暨生物化學系、資訊工程學系、
電機工程學系、通訊工程學系、經濟學系

科目：微積分

第 1 節

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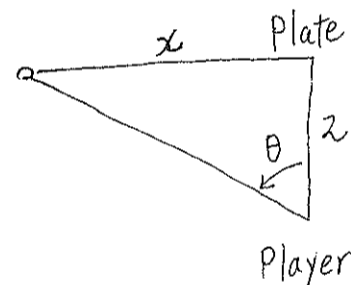
CALCULUS

PART I (70%) - FILL IN THE BLANKS

7% each, NO partial credits.

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- (1) Consider the function $f(x) = \sin(2x) - 2\sin x$, where $x \in]-\pi, \pi[$. $f(x)$ has 3 points of inflection and their x coordinates are given by _____
- (2) The directional derivative of $f(x, y, z) = x^2 + yz$ at $(1, -3, 2)$ in the direction of the path $\mathbf{r}(t) = t^2\mathbf{i} + 3t\mathbf{j} + (1 - t^3)\mathbf{k}$ is _____
- (3) $\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x \sin\left(\frac{1}{1+t}\right) dt =$ _____. (Answer "None" if the limit does not exist.)
- (4) A baseball player stands 2 feet from home plate and watches a pitch fly by. In the diagram, x is the distance from the ball to home plate and θ is the angle indicating the direction of the player's gaze. The instantaneous rate of change θ' of the angle θ at which his eyes must move to watch a fastball with $x' = -130$ ft/s (feet per second) as it crosses home plate at $x = 0$ is _____. Given the fact that humans can maintain focus only when $\theta' \leq 3$, the fastest pitch that you could actually watch cross home plate is _____



- (5) $\int_0^{\frac{\pi}{2}} \frac{\sin(2x)}{(\sin x)^{4/3}} dx =$ _____, $\int_0^1 x \ln(x+3) dx =$ _____
- (6) The value of the infinite series $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$ is _____. (Answer "Diverges" if the series is not convergent.)
- (7) A rocket is launched with a constant thrust corresponding to an acceleration of u ft/s². Ignoring air resistance, the rocket's height after t seconds is given by $f(t, u) = \frac{1}{2}(u - 32)t^2$ feet. Fuel usage for t seconds is proportional to u^2t , so the limited fuel

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capacity of the rocket can be expressed by an equation of the form $u^2t = 10,000$.

The value of u that maximizes the height that the rocket reaches is _____.

- (8) Let Ω be the parallelogram bounded by the $x + y = 0$, $x + y = 1$, $x - y = 0$ and $x - y = 2$. Then $\iint_{\Omega} (x^2 - y^2) dx dy =$ _____.

PART II (30%) - COMPUTATIONAL PROBLEMS

Show all your work. NO CREDITS if only present answers.

- (1) Let f and g be two functions defined and having finite third order derivatives $f'''(x)$ and $g'''(x)$ for all $x \in \mathbb{R}$. If $f(x)g(x) = \pi$, show that the following relations hold at those points where the denominators are nonzero.
- (a) (7%) $\frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)} = 0$.
- (b) (8%) $\frac{f''(x)}{f'(x)} - 2\frac{f'(x)}{f(x)} - \frac{g''(x)}{g'(x)} = 0$.
- (2) A function $F(x, y, z)$ is called homogeneous of degree p if, for all values of the parameter λ , we have the identity

$$F(\lambda x, \lambda y, \lambda z) = \lambda^p F(x, y, z).$$

- (a) (7%) Is the function $G(x, y, z) = x^4 y^2 z \tan^{-1}(z/x)$ homogeneous? If yes, what is the degree? Show all your works.
- (b) (8%) Prove that if $F(x, y, z)$ is differentiable and homogeneous of degree p , then $F(x, y, z)$ satisfies

$$x \frac{\partial F}{\partial x} + y \frac{\partial F}{\partial y} + z \frac{\partial F}{\partial z} = pF.$$