This exam has two parts. Part I consists of blank-filling problems. For each problem in Part I, you must fill in the correct answer for each blank. NO partial credit will be given for a wrong answer.

Part II consists of computational problems. Partial credit may be given.

Part I: Fill in the Following Blanks. (70 points) (填充題)

1. Suppose we know $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$, then what is the value of the integral $\int_{-\infty}^{\infty} e^{-\left(\frac{x^2+6x+9}{4}\right)}\,dx$? Answer: ______

2. Find the directional derivative of the function $f(x,y) = e^{x^2y^3}$ at the point $(1,1)$ in the direction of the unit vector $\mathbf{v} = (\frac{3}{5}, \frac{4}{5})$. Answer: ______

3. If $z = f(x,y)$, where $x = g(t)$, $y = h(t)$, $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, $h'(3) = -4$, $f_x(2,7) = 6$, $f_y(2,7) = -8$, find $\frac{dz}{dt}$ when $t = 3$. Answer: ______

4. Suppose $(1,1)$ is a critical point of a smooth function $z = f(x,y)$. Describe the behavior (i.e., local minimum, saddle point,.....etc.) of f at the point $(1,1)$ if $f_{xx}(1,1) = -4$, $f_{xy}(1,1) = -1$, $f_{yy}(1,1) = -2$. Answer: ______

5. Let $z = x^2 + \sin xy$ be a function of two variables defined on the domain $[-1,1] \times [-1,1]$. Then the surface area of the graph of $z = x^2 + \sin xy$ over its domain is given by the integral formula $\int_{-1}^{1}\int_{-1}^{1} \sqrt{g(x,y)} \cdot dxdy$. What is the function $g(x,y)$? Answer: ______

6. Find the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ where the tangent plane is parallel to the plane $3x - y + 3z = 1$. Answer: ______

7. Evaluate the triple integral $\iiint_B e^{(x^2+y^2+z^2)^{3/2}} \cdot dV$, where B is the unit ball $B = \{(x,y,z) : x^2 + y^2 + z^2 \le 1\}$. Answer: ______

8. Compute the derivative of the function $f(x) = x^{\sqrt{x}}$, $x \in (0,\infty)$ Answer: ______

9. Evaluate the value of the limit $\lim_{x\to 0}\left(\frac{\cos 5x - \cos 8x}{x^2}\right)$. Answer: ______

10. Evaluate the indefinite integral $\int (\ln x)^2\,dx$ Answer: ______

11. Compute $\frac{d}{dx}\int_{\sin x}^{x^2}\sqrt{y^4+1}\cdot dy$. Answer: ________.

12. What is the value of the positive value c if $\sum_{n=2}^{\infty}(1+c)^{-n}=2$. Answer: ________.

13. Find all positive values of b for which the series $\sum_{n=1}^{\infty} b^{\log n}$ converges. Answer: ________.

14. Find all values of c that satisfy the conclusion of the Mean Value Theorem for the function $f(x)=e^{-2x}$ with domain $[0,3]$. Answer: ________.

Part II: Computational Problems. (30 points) (計算題)

1. (15 points) Green Theorem in the plane says that we can use line integral to evaluate the area of a plane region. Use line integral to evaluate the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a, b are positive constants.

2. (15 points) Use Lagrange Multipliers method to find the maximum and minimum values of the function $f(x,y)=x^2y$ subject to the given constraint $x^2+2y^2=6$.