一、單選題 (每題4分, 共16分)

1. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 6 - x^2$. What is the largest area the rectangle can have?

   (A) $4\sqrt{2}$ (B) $6\sqrt{2}$ (C) 10 (D) 12 (E) None of the above

2. Find the area of the region enclosed by the curve $y = x \sin x$ and the x-axis on the interval $4\pi \leq x \leq 5\pi$.

   (A) 8π (B) 9π (C) 4π (D) 5π (E) None of the above

3. Find the limit: $\lim_{x \to 0^+} (e^x + x)^{1/x}$

   (A) 1 (B) e (C) e^2 (D) e^4 (E) None of the above

4. If $f(5) = 4$ and $f'(5) = 2/3$, find $(f^{-1})'(4)$

   (A) 3/2 (B) 2/3 (C) 1/4 (D) 4/5 (E) None of the above

二、複選題(每題6分, 共24分) 請注意: 每題有一個或者一個以上正確答案, 答案完全正確得6分, 否則得0分.

1. Which of the following must be true?

   (A) $\lim_{n \to \infty} \tan\left(\frac{1}{n}\right) = 0$ (B) $\lim_{n \to \infty} \sqrt[n]{3^n + 2^n} = 3$

   (C) $\lim_{n \to \infty} \frac{\tan^{-1} n}{n} = 1$ (D) $\lim_{n \to \infty} \left[\ln(2n^2 + 1) - \ln(n^2 + 1)\right] = 2$

2. Suppose that $f'(x) > 0$ for all $x \in \mathbb{R}$ and that $f(2) = 0$. Let $g(x) = \int_0^x f(t)dt$. Which of the following must be **true**?

(A) The graph of g is increasing on (0,2).

(B) g is a nonnegative continuous function.

(C) The graph of g has a horizontal tangent at $x = 2$.

(D) The graph of g does not have any inflection point.

3. Suppose that series $\sum_{n=0}^{\infty} a_n x^n$ converges when $x = -3$ and diverges when $x = 5$. Which of the following must be **true**?

(A) $\sum_{n=0}^{\infty} a_n$ converges. (B) $\sum_{n=0}^{\infty} a_n(-2)^n$ converges.

(C) $\sum_{n=0}^{\infty} a_n(-5)^n$ diverges. (D) $\sum_{n=0}^{\infty} a_n 6^n$ diverges.

4. Let $f(x,y) = 4 + x^3 + y^3 - 3xy$. Which of the following must be **true**?

(A) f has a local maximum at (0,0). (B) f has a local minimum at (1,1).

(C) At the point $P(1,0)$, f have the maximum rate of change in the direction $\frac{1}{\sqrt{2}}(1,-1)$.

(D) The maximum rate of change at the point $P(1,0)$ is $\sqrt{2}$.

三、填空題(每個空格8分, 共40分)

1. Evaluate the integral $\int_{1/3}^{3} \frac{\sqrt{x}}{x^2+x}\, dx$. $\Rightarrow$ (a)

2. Find the first three nonzero terms in the Maclaurin series for the function $f(x) = e^{-x}\ln(1+x)$. $\Rightarrow$ (b)

3. Find the area of the part of the sphere $x^2 + y^2 + z^2 = 4y$ that lies inside the paraboloid $y = x^2 + z^2$. $\Rightarrow$ (c)

4. Let D be the region enclosed by $y = \dfrac{9}{x^2+9}$, $y = 0$, $x = 0$, and $x = 3$. Find the volume of the solid obtained by rotating about the x-axis. $\Rightarrow$ (d)

5. Let $\mathbf{F}(x,y)$ be the vector field given by $\mathbf{F}(x,y) = \left(\dfrac{-y}{x^2+y^2}, \dfrac{x}{x^2+y^2}\right)$. Calculate $\oint_C \mathbf{F}\cdot d\mathbf{r}$, where C is the unit circle centered at the origin and oriented counterclockwise. $\Rightarrow$ (e)

四、計算題(20 分) 必須有計算過程, 僅有答案而沒有計算過程得0分

1. (10 分) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraints $y - z = 1$ and $z^2 - x^2 = 1$.

2. (10分) Let E be the solid bounded by the paraboloid $z = 6 - x^2 - y^2$ and the surface $z = \sqrt{x^2+y^2}$ and let S be the boundary surface of E, given with positive (outward) orientation. Sketch the solid E and use the **Divergence Theorem** to find $\iint \mathbf{F}\cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x,y,z) = (x^3, y^3, 3xy)$