

一、 填空题(每個空格 7 分, 共 56 分)

1. If  $f(x) = \exp(g(x))$ , where  $g(x) = \int_0^{\sin(\pi x)} \sqrt{1+t^2} dt$ , find  $f'(1)$ . (a)           

2. Find the interval of convergence of the series  $\sum_{n=0}^{\infty} \frac{(x-3)^n}{n^2-1}$ . (b)           

3. For what value of  $C$  does the equation  $\ln x = Cx^3$  have exactly one solution? (c)           

4. Find the linearization  $L(x, y) = Ax + By + C$  of the function  $f(x, y) = x\sqrt{y}$  at the point  $(-5, 4)$ . (d)           

5. Find the arc length of the curve  $y = \frac{x^2}{2}$ ,  $x \in [0, 1]$ . (e)           

6. The base of a solid is the ellipse  $x^2 + 4y^2 = 4$ , and every parallel cross sections perpendicular to the  $x$ -axis are equilateral triangles (等邊三角形). Find the volume of the solid. (f)           

7. Find the area of the region enclosed by the curve  $r^2 = 4 \sin 2\theta$ . (g)           

8. Let  $S$  be the surface of the solid  $E$  that lies above the cone  $z = \sqrt{x^2 + y^2}$  and below the sphere  $x^2 + y^2 + z^2 = z$  and  $\mathbf{F}(x, y, z) = (z, y, \sin(x+y))$ . Evaluate  $\iint_S \mathbf{F} \cdot d\mathbf{S} =$   
(h)

# 國立中正大學101學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系

學系別：資訊工程學系、電機工程學系、機械工程學系 科目：微積分

化學工程學系

第 1 節

第 2 頁，共 2 頁

二、計算題(44分，這部分必須有完整計算過程，僅有答案而沒有計算過程得0分)

1. (4+4+4=12分) Sketch the solid whose volume is given by  $\int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} f(x, y, z) dy dx dz$ .

Then, rewrite this iterated integral in the following orders:

(a)  $\iiint f(x, y, z) dx dz dy$       (b)  $\iiint f(x, y, z) dy dz dx$

2. (5+5=10分) Please complete the following steps to compute the improper integral

$$\int_0^{\infty} e^{-x^2} dx:$$

(a) First of all, compute the double integral  $\iint_{R^2} e^{-x^2-y^2} dA$

- (b) Apply Fubini's theorem and symmetric property of the function  $e^{-x^2}$  to find the

$$\text{integral } \int_0^{\infty} e^{-x^2} dx:$$

3. (4+2+6=12分) (a) Derive the formula:  $\int_0^1 x^n (\ln x)^k dx = \frac{-k}{n+1} \int_0^1 x^n (\ln x)^{k-1} dx$

(b) Derive the formula  $\int_0^1 x^n (\ln x)^n dx = \frac{(-1)^n n!}{(n+1)^{n+1}}$

- (c) Use the formula  $x^x = e^{x \ln x}$  and the Maclaurin series for  $e^x$  to derive the formula

$$\int_0^1 x^x dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^n}$$

4. (10分) Let  $f(x, y) = 8xy - 2x - 4y + 5$ . Find the absolute minimum value of the function  $f(x, y)$  on the set  $D$ , where  $D$  is the region bounded by the parabola  $y = x^2$  and the line  $y = 4$ .