

國立中正大學100學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系

學系別：資訊工程學系、電機工程學系、機械工程學系

科目：微積分

化學工程學系、通訊工程學系

第 1 節

第 1 頁，共 2 頁

一、填充題(每個空格 7 分，共 35 分)

1. Find the limit: $\lim_{x \rightarrow 0^+} (1 + 3x)^{\csc x} = \underline{\hspace{2cm}}$

2. Evaluate the integral: $\int \frac{dt}{t^{1/2} + t^{3/2}} = \underline{\hspace{2cm}}$

3. Find the volume of the solid bounded by the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ and the plane $z = 0$ and $z = 3x + y$. $\underline{\hspace{2cm}}$

4. Apply the Green's theorem to find the area of the region enclosed by the curve:
 $\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{j}$, $0 \leq t \leq 2\pi$. $\underline{\hspace{2cm}}$

5. Find the surface integral $\iint_S y dS$, where S is the surface $x - y^2 - z = 0$,
 $0 \leq y \leq 2$, $0 \leq z \leq 1$. $\underline{\hspace{2cm}}$

二、複選題(每題 7 分，共 35 分。請注意：每題至少有一個或者一個以上正確答案，答案完全正確得 7 分，否則得 0 分)。

1. Consider the graph of the function $f(x) = \frac{x^2 - x - 2}{x - 3}$. Which of the following must be true?

- (A) There is a vertical asymptote at $x = 3$. (B) $f(5)$ is a relative minimum.
(C) $f(1)$ is a relative minimum. (D) The graph does not have any inflection point.

2. Which of the following must be true?

(A) The power series $\sum_{k=1}^{\infty} \frac{x^{2k-1}}{(2k-1)}$ has radius of convergence $r = 1$.

(B) The interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-1)^{2k-1}}{(2k-1)}$ is $(0, 2]$.

(C) The power series $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$ converges for all $x \in \mathbb{R}$.

(D) The power series $\sum_{k=0}^{\infty} n^n x^n$ diverges for all $x \in \mathbb{R}$.

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第 2 頁，共 2 頁

3. Which of the following must be true?

(A) The sequence $\{\frac{n}{e^n}\}$ decreases for $n \geq e$. (B) The series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k!}$ converges absolutely.

(C) The series $\sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k}$ converges conditionally. (D) The series $\sum_{k=1}^{\infty} \frac{1}{k \cdot \ln k}$ diverges.

4. Set $f(x, y) = \begin{cases} \frac{3x^2}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$ Which of the following must be true?

(A) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist. (B) $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$.

(C) $f_x(0, 0)$ does not exist. (D) $f_y(0, 0) = 0$.

5. Let k be a constant and consider the function $f(x, y) = x^2 - kxy + y^2$. Which of the following must be true?

(A) $\nabla f(0, 0) = \vec{0}$ for any k . (B) f has local maximum at $(0, 0)$ if $k = 0$.

(C) f has local minimum at $(0, 0)$ if $k = 1$. (D) f has saddle point at $(0, 0)$ if $k = 3$.

三、計算題(30分，這部分必須有計算過程，僅有答案而沒有計算過程得0分)

1. (10分) Sketch the region Ω bounded by the x -axis, y -axis and the function

$$f(x) = \begin{cases} \frac{\sin x}{x}, & 0 < x \leq \pi, \\ 1, & x = 0. \end{cases}$$

Find the volume of the solid generated by revolving the region Ω about the y -axis.

2. (3+7=10分) Evaluate $\iint_{\Omega} \left(\frac{x^2 - xy + y^2}{2}\right) dA$ where Ω is the region enclosed by the ellipse

$$x^2 - xy + y^2 = 2 \text{ and using the transformation } \begin{cases} x = \sqrt{2}u - \sqrt{2/3}v, \\ y = \sqrt{2}u + \sqrt{2/3}v. \end{cases}$$

3. (10分) Evaluate $\int_C \vec{F} \cdot d\vec{x}$, where $\vec{F} = z^2\mathbf{i} + y^2\mathbf{j} + x\mathbf{k}$ and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.